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**ENHANCEMENT OF HEIGHT SYSTEM FOR MALAYSIA USING  
SPACE TECHNOLOGY :**

**THE STUDY OF THE DATUM BIAS INCONSISTENCIES  
IN PENINSULAR MALAYSIA**

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## **ABSTRACT**

The algorithm for orthometric height transfer using GPS has been widely presented. Its practical limitations are mostly due to datum bias inconsistencies and lack of precise geoid. In most applications, datum biases are assumed to be systematic over short baselines and therefore could be eliminated by differential heighting techniques. In this study, optimal algorithms were investigated to model biases between local vertical datum in Peninsular Malaysia and the datums implied by EGM96, OSU91A and the regional Gravimetric Geoid in South\_East Asia.

The study has indicated that local vertical datum is not physically parallel to the datums implied by the above geoids. The shift parameters between the datums implied by the GPS/leveling data, and the EGM96, OSU91A and the gravimetric datums are about – 41cm, -54 cm and – 8 cm respectively. Also the maximum tilts of the planes fitting the residual geoids above these datums relative to GPS/Leveling datum are of the order of 36, 51 and 33 centimeters per degree. It is therefore necessary to take into account the effect of inconsistent datum bias particularly for baseline height transfer.

The level of accuracy achieved by the bias corrected relative orthometric height differences of the EGM96, OSU91A and the gravimetric geoid models combined with GPS/leveling data for baseline lengths up to 36 km, is sufficient to replace the conventional tedious, time consuming ordinary leveling technique for rapid height transfer for land surveying and engineering applications.

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## **1.0 INTRODUCTION**

### **1.1 Background**

The need to have a unified vertical reference frame becomes so apparent in the era of information technology. The reliability of information requiring geographical position needs the points on the earth to be defined uniquely and accurately. It is the role of geodesist to provide the reference system which is capable to support the need for present and future advancements in mapping science and information technology.

Unification of vertical reference systems requires both geometrical and physical connections in the study area. Precise geometrical connection is provided from the regional GPS network while the physical relationship is to be provided from the regional or global precise geoid covering the whole of study area.

The algorithm for orthometric height transfer using GPS has been widely presented and used. Its practical limitations are mostly due to datum bias inconsistencies and lack of precise geoid.

For the past 100 years or so, traditional spirit leveling has been a technique of choice in the determination of orthometric heights. It is simple, the operation is effective, the method has remained basically unchanged, and yet it can achieve a remarkable precision. However, the observational time is too lengthy, making it a slow, labour-intensive, painstaking and costly operation. It is also a line operation whereby points whose heights are required need to be interconnected by a series of leveling lines. This makes levelling operation prone to many systematic errors which are difficult to detect and eliminated. Thus in recent times, many efforts were made to develop alternative techniques and technologies to levelling to suit current needs.

Nowadays, the many benefits offered by space based measurements systems such as the Global Positioning System (GPS) have made it a suitable alternative over traditional levelling. Orthometric height determination using GPS is one of the possible applications that is gaining popularity. In recent times, the use of GPS for surveying, engineering and mapping applications is expanding at an astounding rate. This has prompted many countries, including Malaysia to not only upgrade their existing reference systems to be GPS-compatible, but also to seriously look into the potential use of GPS in heighting.

However, there are several problems associated with the use of GPS in vertical positioning. First GPS gives elevations above a reference ellipsoid, World Geodetic System 1984 (WGS84). It gives heights that cannot be used directly with traditional orthometric height datums especially in determining the directions of water flow. Neither could the ellipsoidal heights be directly incorporated into the gravity based height systems.

Therefore, GPS-based levelling technique needs to use ellipsoidal height differences as well as the geoidal height differences in order to obtain orthometric height differences. In order to derive GPS height with respect to this vertical datum, the geoid-ellipsoid separation needs to be deducted. As such, orthometric heights can only be obtained from the knowledge of ellipsoidal height from GPS and geoidal heights from a geoid model.

As most engineering and mapping activities are referenced to an orthometric height surface, GPS users requiring orthometric heights need to perform geoid modeling.

This research explores the potential use of GPS in height surveys. First it outlines the principles and applications of modelling the geoid over Peninsular Malaysia. Particular attention is directed towards the use of EGM96 global geopotential model and gravity computation using the fast Fourier transform (FFT) technique where an analysis of the geoids in the region is performed. Following this, both geoid heights are compared with the GPS-levelling geoid. The second part of the research describes the about the detailed

study on the datum bias. The test area is Perak. The third part of the research describes a GPS survey carried out on bench marks in a test area. The aim is to investigate the achievable accuracy of GPS heighting in Johor. This is to enable a comparison to be made between GPS-derived orthometric heights and spirit levelled heights of bench marks. This work will demonstrate GPS as a viable technique of transforming GPS ellipsoidal heights to orthometric heights using a local geoid model. Graphical and statistical comparisons are also made with GPS-derived geoid model to determine the effectiveness of GPS height survey.

## **1.2 Objectives**

Main objectives of this research are :

- To model vertical datum bias
- To evaluate accuracy of long baseline height determination
- To evaluate status of NGVD (initial study)
- To investigate the application of GPS Heighting

## 2.0 RESEARCH METHODOLOGY

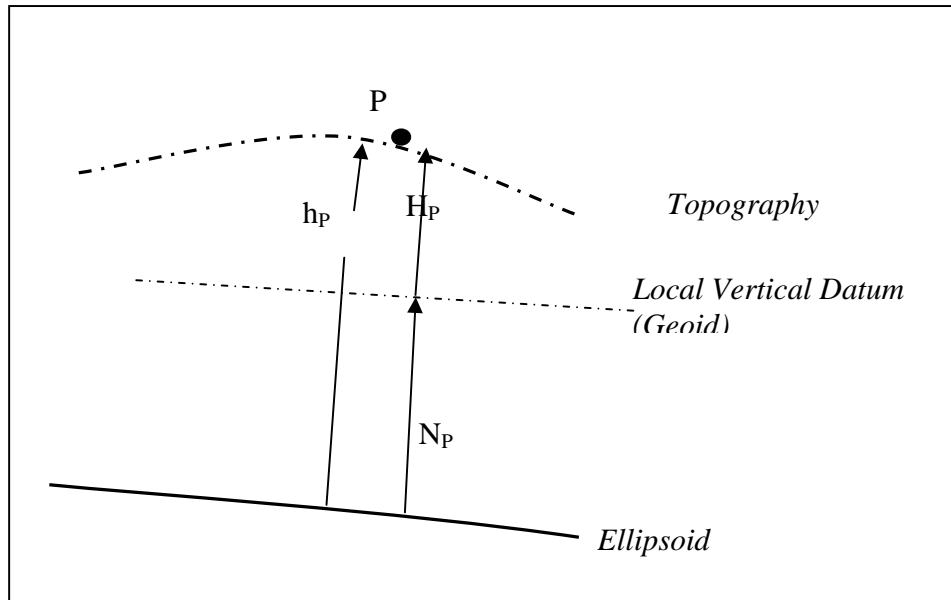
### 2.1 Background Theory

#### 2.1.1 GPS Heighting

Due to the fact that ellipsoidal heights are geometric values and orthometric heights are physical values reflecting local variations in gravity as well as changes in topography, the conversion from ellipsoidal to orthometric height requires a geoid height model. The process of converting ellipsoidal heights to orthometric heights or ellipsoidal height differences to orthometric height differences from a known geoid model is known as GPS heighting. In this study, four different geoid models were considered for this investigation. These are the EGM96, OSU91A , gravimetric and the GPS/leveling geoids.

##### 2.1.1.1 Absolute GPS Heighting without Datum Bias Term

When dealing with absolute GPS heighting without datum bias term, we are assuming that, the local vertical datum is coincident with the geoid as shown in Fig. 1.



**Figure 1** Absolute GPS heighting without datum bias term

The orthometric heights above EGM96, OSU91A and the gravimetric geoids are computed from:

$$H_P = h_P - N_P \quad (1)$$

The orthometric height differences ( $\delta H$ ) of the EGM96, OSU91A and the gravimetric models relative to the local vertical datum are computed from:

$$\delta H_P = H_P^{BM} - H_P = H_P^{BM} - h_P + N_P \quad (2)$$

Where;

$H^{BM}$  : is the true orthometric height from leveling at the Bench Marks

### **2.1.1.2 Relative GPS Heighting without Datum Bias Term**

However due to the fact that in surveying and geodetic applications, differential (relative) GPS is used to provide ellipsoidal height differences with respect to a fixed base station, the absolute orthometric heights of equation (1) may not be relevant. Instead the change in orthometric height over a GPS baseline (A to B) is determined by using the corresponding change in geoid height. Also when dealing with relative GPS heighting without datum bias term, we are assuming that, the local vertical datum is coincident with the geoid.

Relative orthometric, ellipsoidal and geoidal height differences of the EGM96 model, OSU91A model, gravimetric model and the GPS/leveling data relative to the fixed station are computed from:

$$\Delta H_{AB} = H_B - H_A = \Delta h_{AB} - \Delta N_{AB} \quad (3)$$

Residual orthometric height differences of the EGM96, OSU91A and the gravimetric model relative to the local vertical datum are computed from:

$$\delta \Delta H_{AB} = \Delta H_{AB}^{BM} - \Delta H_{AB} \quad (4)$$

with:

$$\Delta H_{AB}^{BM} = H_B^{BM} - H_A^{BM}$$

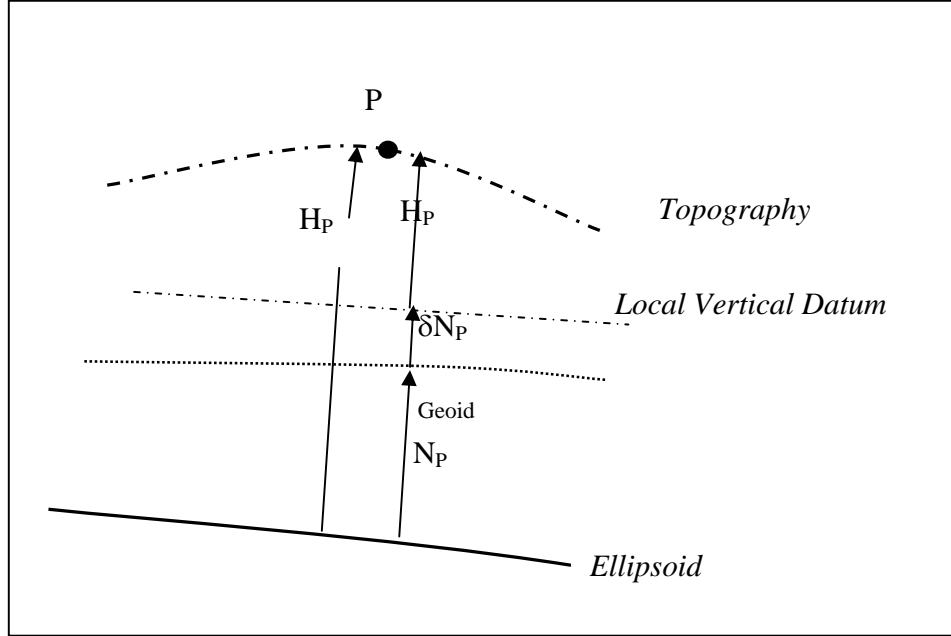
These relative orthometric height differences are scaled by their corresponding distances and expressed in parts per million (ppm) as:

$$\text{Error in ppm} = \delta \Delta H_{AB} / S_{AB} \quad (5)$$



### 2.1.1.3 Absolute GPS Heighting with Datum Bias Term

When dealing with absolute GPS heighting with datum bias term, the assumption is that the local vertical datum does not coincide with and may not be parallel to the geoid as shown in Fig. 2.



**Figure 2** Absolute GPS heighting with datum bias term

The bias corrected geoidal heights referred to the EGM96, OSU91A and gravimetric models were computed from:

$$N_P^* = N_P - \delta N_P \quad (6)$$

Also the bias corrected orthometric heights above EGM96, OSU91A and the gravimetric geoid models are computed as follows:

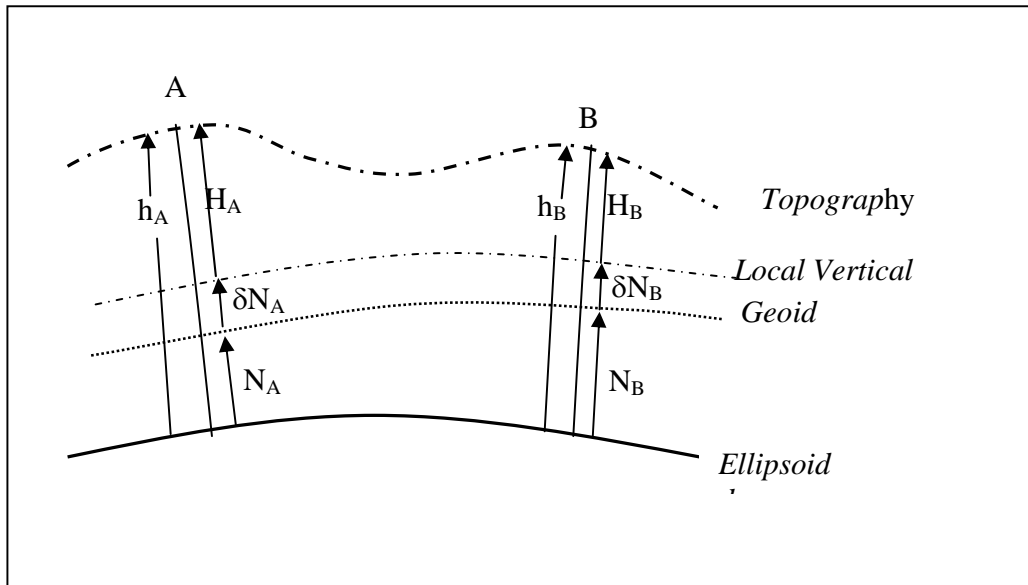
$$H_P^* = h_P - N_P^* = h_P - N_P + \delta N_P \quad (7)$$

The bias corrected orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum (GPS/leveling datum) are computed from:

$$\delta H_P^* = H_P^{BM} - H_P^* = H_P^{BM} - h_P + N_P - \delta N_P \quad (8)$$

#### 2.1.1.4 Relative GPS Heighting with Datum Bias Term

Also when dealing with relative GPS heighting with datum bias term, the assumption is that, the local vertical datum does not coincide with and may not be parallel to the geoid as shown in Fig. 3.



**Figure 3** Relative GPS heighting with datum bias term.

Relative bias corrected orthometric and geoidal height differences of the EGM96l, OSU91A, and the gravimetric models relative to the fixed station are computed from:

$$\Delta H_{AB}^* = H_B^* - H_A^* = \Delta h_{AB} - \Delta N_{AB}^* \quad (9)$$

With:

$$\Delta N_{AB}^* = \Delta N_{AB} - \delta N_{AB}$$

$$\Delta \delta N_{AB} = \delta N_B - \delta N_A$$

Residual bias corrected orthometric height differences of the EGM96, OSU91A and the gravimetric models relative to the local vertical datum are computed from:

$$\delta \Delta H_{AB}^* = \Delta H_{AB}^{BM} - \Delta H_{AB}^* \quad (10)$$

Also these relative orthometric height differences are scaled by their corresponding distances and expressed in parts per million (ppm) as:

$$\text{Error in ppm} = \delta \Delta H_{AB}^* / S_{AB} \quad (11)$$

### 2.1.2 Vertical Datum Bias Inconsistencies

The GPS/leveling- derived geoidal heights should be combined with gravimetric data to determine the best possible geoid. This is simply done by using the GPS/leveling data to shift and tilt the gravimetric geoid to get the best possible match in a region. Mean Sea Level heights at different locations does not lie on the same equipotential surface, vertical datums of the world have inconsistent reference surfaces at the  $\pm 2\text{m}$  discrepancy level (Rapp and Balasubramania ,1992). The estimation of orthometric heights from the combination of ellipsoidal heights derived from GPS and existing geoid models in absolute or relative mode is subject to the following error sources:

### **2.1.2.1 Errors in GPS Ellipsoidal Heights**

The main error source in GPS ellipsoidal heights were found to occur depending on the mode (point positioning, differential, kinematics, etc) in which the GPS survey is conducted, due to the resolution of the GPS observable ( C/A, L1, L1&L2,..tc) used ( Featherstone et al 1998). In addition to the mode of GPS survey and the observables used, other errors inherent to GPS surveying affects the GPS heights. These are :

- Vertical dilution of precision (VDOP).
- Satellite ephemeris and GPS baseline length
- The atmosphere
- Multipath
- Antenna orientation and phase center
- Measurement of antenna height
- Fixing the integer ambiguities

### **2.1.2.2 Errors in Geoid Determination**

The determination of geoid undulation indicates four main error sources in undulation estimation.

- Errors associated with the gravity data in the spherical cap surrounding the computation point.
- Errors associated with the gravity anomalies being given at discrete locations (mean values) instead of as a continuous function.
- Commission errors associated with the potential coefficients of the geopotential model used in the combination process.
- Omission errors associated with the neglected potential coefficients above degree  $N_{MAX}$ .

- Errors associated with topographical attraction effects on the gravity anomalies
- Errors due to indirect effects on the computed geoid
- Spherical approximation errors

### **2.1.2.3 Errors in Orthometric Heights**

For the assessment of the accuracy of a geoid model in a survey area, orthometric heights derived from spirit leveling are needed. These heights are subject to systematic and random errors inherent in most local and regional leveling networks. These errors could be due to the following:

- Instrumental errors
- Atmospheric errors
- Data reduction errors
- Improper leveling network adjustments

### **2.1.2.4 Vertical Datum Errors**

- Non-parallelism of equipotential surfaces
- Vertical deformation errors
- Vertical datum definition errors

### 2.1.3 Modeling Vertical Datum Biases

Since a local vertical datum and a given geoid are theoretically parallel surfaces, datum biases (inconsistencies) should, at least form a uniform surface that can be represented by a plane surface. The geoidal height differences computed at the GPS stations from EGM96, OSU91A and the gravimetric geoid models relative to the local vertical datum, are subject to errors and biases due to several factors as mentioned above. These biases can be reduced or absorbed by fitting a plane surface to these residuals as follows:

$$\delta N(\phi, \lambda) = c_1 \rho \Delta \phi + c_2 \nu \cos \phi \Delta \lambda + c_0 \quad (12)$$

$$\rho = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}} \quad (13)$$

$$\nu = \frac{a}{(1 - e^2 \sin^2 \phi)^{\frac{1}{2}}} \quad (14)$$

$$\Delta \phi = \phi - \phi_0$$

$$\Delta \lambda = \lambda - \lambda_0$$

Where;

$\rho$	:	meridian radius of curvature
$\nu$	:	prime vertical radius of curvature
$\phi$	:	geodetic latitude of the station
$\lambda$	:	geodetic longitude of the station
$\phi_0$	:	geodetic latitude of the origin
$\lambda_0$	:	geodetic longitude of the origin

$c_1$	:	north-south tilt of the plane
$c_2$	:	east-west tilt of the plane
$c_0$	:	shift between the local vertical datum and the gravimetric datum
$a$	:	semi-major axis of the reference ellipsoid
$e$	:	first eccentricity of the ellipsoid

Our interest lies in the maximum tilt of this plane with respect to some horizontal plane on which the residual would be random, as well as the direction of the maximum tilt. The unit of tilt is distance per degree. The maximum tilt (T) of the plane fitting the residuals and the azimuth (A) with respect to north at the origin point of the plane are given as:

$$T = \sqrt{a^2 + b^2} \quad (15)$$

$$A = \arctan(b/a) \quad (16)$$

From the data of Table 3, the residual differences between GPS/leveling geoid and the EGM96, OSU91A and the gravimetric geoid are computed. Considering these differences as observations, the datum bias parameters  $C_0$ ,  $C_1$  and  $C_2$  are estimated from a least squares. Also the maximum tilt (T) and the direction of maximum tilt (A) are computed from equation (9) and equation (10) respectively. The datum biases between the local vertical datum and EGM96, OSU91A and the gravimetric geoid at the twenty-seven GPS stations computed from a least squares .

#### 2.1.4 Geoid Computation Method

As mentioned, the geoid can be defined as the gravity equipotential surface which best approximates mean sea level over the whole earth and as such it also acts as the datum for orthometric height system (Kuang *et. al.*, 1996). Since the geoid reflects the earth's gravity field, its shape is irregular and undulating. So, a reference surface of an ellipsoid, a regular mathematical figure, is selected to approximate the geoid surface. The geoidal height and the deflection of vertical describe the relationship between the two surfaces and they can be determined by various approaches. Many advances in the field of geoid computations, both in terms of theory and algorithm development have occurred in the past decade. These include the gravimetric method, the astro-geodetic method, satellite dynamic solution for potential coefficients and direct determination from 3-D geocentric coordinates and orthometric heights (Vanicek & Krakiwsky, 1986; Torge, 1980). Each of these approaches has its advantages and disadvantages.

A gravimetric geoid model provides a very high resolution and local accuracies. However, one major drawback that it has is that it is subject to long-wavelength systematic effects. With the recent developments in methodology and techniques, older geoids based on astro-geodetic have now been superseded by global geoids evaluated using gravimetric methods. Nowadays GPS technique allows the determination of geoid heights  $N$  of points of which height is known from levelling  $H$ . It provides geocentric position of the point with  $h$  as the height above a reference ellipsoid such as WGS-84 or GRS80 by the relationship  $N = h - H$ .

The gravimetric determination of the geoid at any point of geographical coordinates ( $\phi$ ,  $\lambda$ ) essentially relies on the classical Stokes' Integral (Heiskanen & Moritz, 1967). This amounts to evaluating the integral for the sphere as follows:

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} S(\psi) \Delta g \, d\sigma \quad (17)$$

where  $R$  is the mean radius of the Earth,  $\gamma$  is the normal gravity on the GRS80 reference



ellipsoid at the geodetic latitude of the computation point,  $\Delta g$  is the free-air gravity anomaly,  $d\sigma$  is the element of surface area for integration on the sphere and  $\psi$  is the angular distance between the point of computation and  $d\sigma$ , and  $S(\psi)$  is the Stokes' function of the angular distance  $\psi$  given by:

$$S(\psi) = \frac{1}{\sin \frac{\psi}{2}} - 4 - 6 \sin \frac{\psi}{2} + 10 \sin^2 \frac{\psi}{2} - \left( 3 - 6 \sin^2 \frac{\psi}{2} \right) \ln \left( \sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2} \right) \quad (18)$$

The gravimetric geoid separation  $N$  can be computed by determining the long- and medium-wavelength contributions  $N_{\text{egm}}$  from the global geopotential model (GGM) coefficients of EGM96, the short- and part of the remaining medium-wavelength effect  $N_{\Delta g}$  from terrestrial gravity data and associated height information  $N_h$  from a digital elevation model (DEM). This procedure is represented by the following relationship, such that:

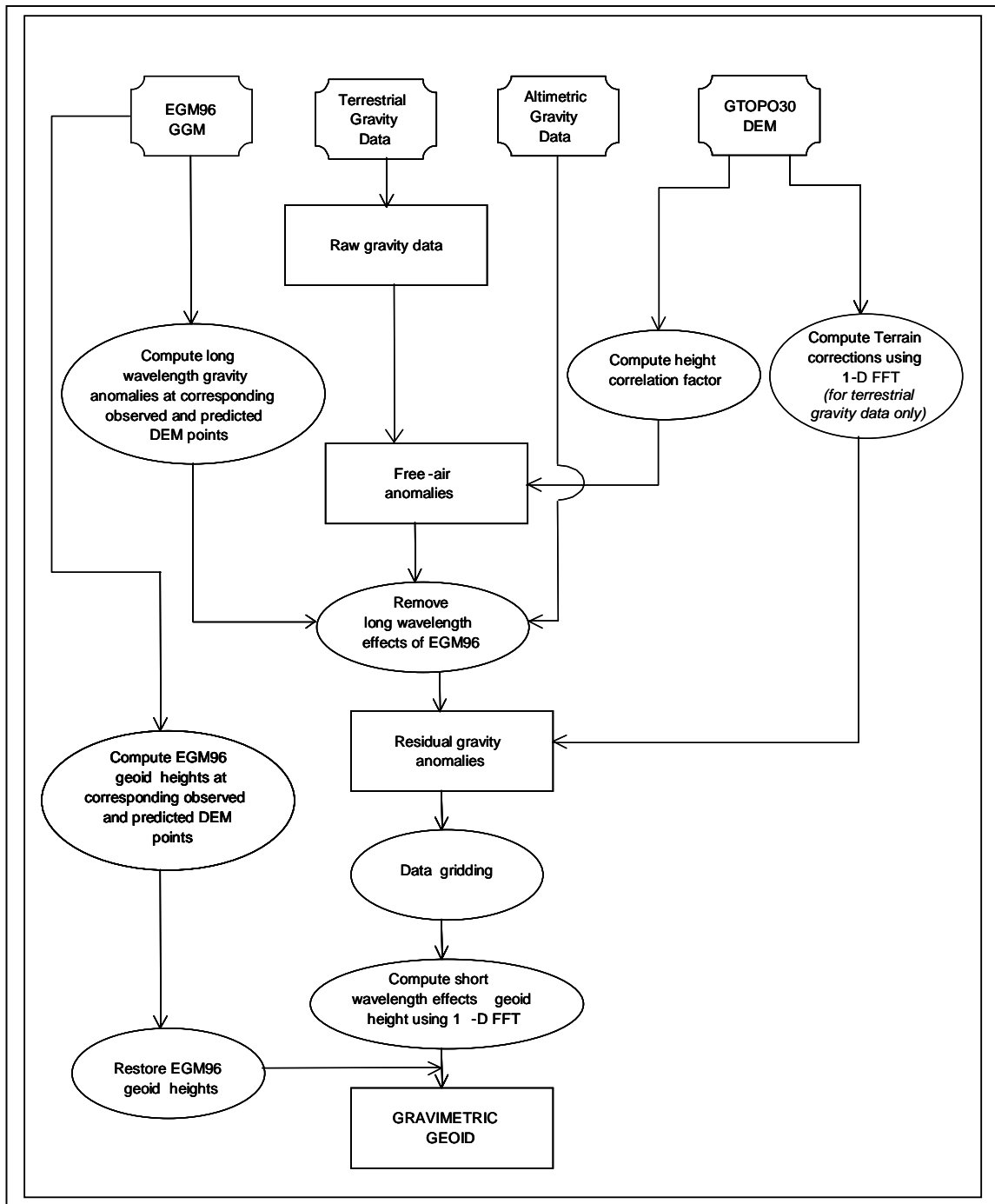
$$N = N_{\text{egm}} + N_{\Delta g} + N_h \quad (19)$$

The method used in this study breaks the gravity field into the three component and solve them separately. The principles of the computation procedure are as follows:

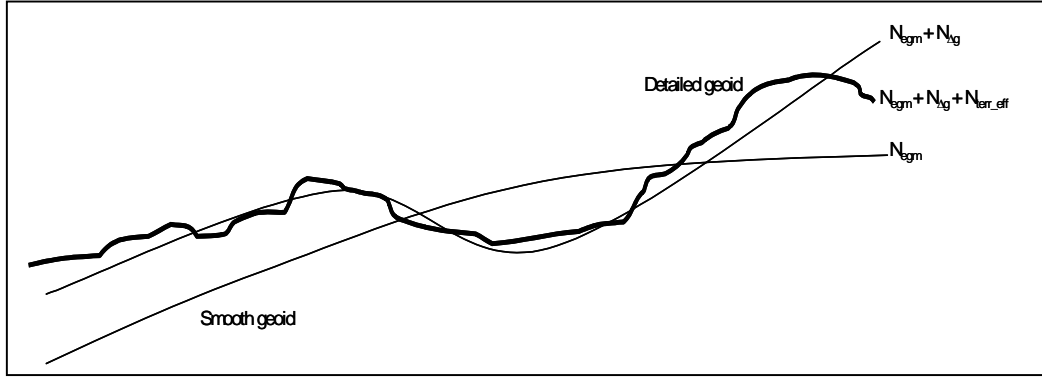
- Long wavelength values are determined by global geopotential model;
- Topographical masses are removed and restored mathematically through reductions of anomalies; and
- Short wavelength gravity anomalies are interpolated by fast Fourier transform of the Stoke's function.

A flowchart depicting the determination of the three signals and the generation of a gravimetric geoid is given in Figure 4. Figure 5 demonstrates the three contributions of the long, medium and short wavelengths of the Earth's gravity field.  $N_{\text{egm}}$  represents long wavelength geoid features of the EGM96 geopotential model over distances in the order

of 100 km. Due to the scarcity of global gravity data coverage, the long wavelength components are less accurate. This deficiency is compensated by satellite-derived gravity data. Since the strength of the gravity field degrades with distance from the geocentre, only the low frequency (ie. the long wavelength) component  $N_{\text{egm}}$  can be detected. It changes very smoothly with magnitude in metres. On the other hand, terrestrial gravity data gives detailed local information about the short wavelength contribution  $N_{\Delta g}$  of the Earth's gravity field.  $N_{\Delta g}$  has the order of magnitude of in decimeters. It represents regional geoid features with wavelengths of 20 to 100 km.  $N_h$  is introduced by the topography as a contribution of the DEM. It represents the wavelength features below 20 km and changes rapidly with magnitudes in centimeters.



**Figure 4** A flowchart to compute a gravimetric geoid via FFT and using the R-C-R approach



**Figure 5** Contributions of Global Gopotential Model, terrestrial gravity data and height information to gravimetric geoidal separation (adapted from Sideris, 1993)

From Equation (17), it can be seen that the approach by Stokes in the determination of a geoid model requires a global gravity coverage. Practically, this is impossible to achieve, not only because of many areas of the world still has no gravity data but also due to the fact that many countries still treat the data as highly classified piece of information. These difficulties can be avoided by using the “remove-compute-restore” technique (Schwarz *et. al.*, 1990), which is one of the most commonly routine in computing regional gravimetric geoid nowadays. This technique requires the removal of spherical harmonic model (SHM) gravity anomalies before computing the Stokes’ Integral, and the restoration of SHM geoid undulations following it.

The “remove-compute-restore” approach exploits the linearity of Equation 19 where the computation of a local geoid is split into three distinct parts:

- i. First, *remove* from the terrestrial free-air gravity anomalies  $\Delta g_{fa}$ , the total effect of the EGM96 global geopotential model (the long wavelength contribution),  $\Delta g_{egm}$  and the terrain (short wavelength information),  $\Delta g_h$  to derive the residual gravity anomalies,  $\Delta g_{rga}$ . This is carried out by first interpolating the anomalies from EGM96 and then subtracting them from the observed gravity anomalies at each gravity observation points, such that:

$$\Delta g_{rga} = \Delta g_{fa} - \Delta g_{egm} - \Delta g_h \quad (20)$$

From EGM96, a reference gravity anomaly  $\Delta g_{egm}$  is computed using:

$$\Delta g_{egm} = \frac{GM}{R^2} \sum_{n=2}^{360} (n-1) \sum_{m=0}^n [C_{nm} \cos m\lambda_p + S_{nm} \sin m\lambda_p] \times P_{nm}(\sin \phi_p) \quad (21)$$

where  $G$  is the Newton's gravitational constant,  $C_{nm}$  and  $S_{nm}$  are the fully normalised spherical harmonic coefficients obtained from EGM96 geopotential model to degree and order 360,  $P_{nm}(\sin \phi_p)$  is the fully normalised associated Legendre polynomial,  $n$  and  $m$  are the degree and order of EGM96 respectively and 360 is the maximum degree of EGM96.

- ii. Next, *compute* the residual geoidal heights,  $N_{\Delta g}$  via FFTGEOID software (Li, 1994), based on 1D-FFT technique using the derived residual gravity anomalies,  $\Delta g_{rga}$

By applying the Stoke's integral, it is now possible to compute the gravimetric geoid for Peninsular Malaysia. However, the formula represents the surface integral over the whole area of interest, it will result in constraints in terms of computing time. One way to overcome this problem is to employ the FFT approach.

The 1D-FFT or one-dimensional fast Fourier transform is one of the most recent and popular approach for the evaluation of Stoke's integral. The integration is done in an area of interest, instead of the whole surface of the Earth as required by the Stoke's integral. FFT technique requires fully reduced gridded gravity anomalies. The gridding algorithm employed in this study used a method of Kriging.

FFT technique evaluates the discrete spherical Stokes integral parallel by parallel without any approximation, along each parallel of latitude. In this technique, a rectangular zone of geoidal heights is produced by integrating the Cartesian rectangular zone (x,y) of gravity anomalies within the following geographical boundaries:  $0^\circ 00' N \leq \phi \leq 8^\circ 00' N$  and  $99^\circ 00' E \leq \lambda \leq 105^\circ 00' E$ . FFT technique expedites many of the computations used in modeling the gravity field very efficiently using gridded digital terrain data and mean gravity values. In order to reduce spectral leakage and to eliminate circular convolution effects, a 100% zero padding is applied on the east and west edges of the gravity anomaly input grid.

The Stoke's equation is written as:

$$N_{\Delta g}(\varphi_p, \lambda_p) = \frac{R\Delta\varphi\Delta\lambda}{4\pi\gamma} \sum_{i=1}^M \left\{ \sum_{j=1}^N S(\varphi_p, \varphi_i, \lambda - \lambda_j) \Delta g(\varphi_i, \lambda_j) \cos \varphi_i \right\} \quad (22)$$

The terms in the brackets contain 1D discrete convolution with respect to  $\lambda$  which can then be evaluated by the 1D FFT technique. Following this, the 1D-FFT expression is as follows:

$$N_{\Delta g}(\varphi_p, \lambda_p) = \frac{R\Delta\varphi\Delta\lambda}{4\pi\gamma} F^{-1} \left\{ \sum_{\varphi_Q=\varphi_1}^{\varphi_{\max}} F[S(\psi_{PQ})] F[\Delta g(\varphi_Q, \lambda_Q) \cos \varphi_Q] \right\} \quad (23)$$

where the operators  $F$  and  $F^{-1}$  denote the direct and inverse 1D discrete Fourier transforms respectively,  $\Delta\varphi$  and  $\Delta\lambda$  are the used latitudinal and longitudinal grid spacing, and  $\varphi_1$  and  $\varphi_{\max}$  are the southern and northern grid boundaries respectively.

Thus, the geoid undulation for all points on a parallel can be obtained.

- iii. And finally, the last step is to *restore* back the effect of the EGM96 global geopotential model,  $N_{egm}$  (the long wavelength contribution) to the residual geoidal heights,  $N_{\Delta g}$  and add the terrain effect term,  $N_H$  (computed from digital elevation model) to form the final geoid undulations.

Again from EGM96, the reference geoidal undulation  $N_{egm}$  or the long-wavelength geoid component was also made on a 2' x 2' grid within the geographical boundaries specified above. This is also the grid configuration in which the final MYGeoid02 geoid heights are given.  $N_{egm}$  was computed using:

$$N_{egm} = R \sum_{n=2}^{360} \sum_{m=0}^n \left[ C_{nm} \cos m\lambda_p + S_{nm} \sin m\lambda_p \right] P_{nm} \sin \phi_p \quad (24)$$

## **2.2 Research Area**

This research has been mainly conducted in three areas which each area has its own objectives. The areas are:

- Peninsular Malaysia GPS Network . This area research aims to model the geoid and datum bias over Peninsular Malaysia.
- The State of Perak. This area research aims to be detailed study of datum bias.
- The State of Johor. This area research aims to investigate the accuracy of GPS heighting.

### **2.2.1 Preliminary Geoid Computation Over Peninsular Malaysia**

The geoid is most commonly defined as the hypothetical equipotential surface of the Earth's gravity field, which would closely coincide with undisturbed mean sea level (MSL). Undisturbed MSL would only exist if the oceans were acted on by the Earth's gravitational field whilst, ignoring other forces such as tides, ocean currents and winds. Geoid determination remains as one of the basic task of ongoing researches in physical geodesy. This is because the majority of measurements are referred to Earth's gravity field. The geoid also reflects variations in the gravity field. Traditionally its surface has served as the fundamental reference for orthometric height and its differences, gravity potentials and other vertical heights.

The past decade had seen a renewed interest in the determination of geoid on global, regional and local scales. Great strides have been achieved both in terms of theory, techniques and algorithm development in the quest to achieve precise geoids. There has also been an increase in worldwide coverage of terrestrial gravity collection. In Peninsular Malaysia, the national gravity database is continuously being updated. The geographical distribution of gravity data over the land areas has not only been widespread but also densified substantially over the last decade. The number of gravity points was less than 2000 in 1989 (Kadir *et. al.*, 1989) and by 1995 it had increased to more than



5000 points (Ses & Majid, 1998). In 2000, the gravity database at DSMM has a total number of over 10000 points. However, up to now, there is no precise geoid available yet for Peninsular Malaysia and its surrounding seas, although attempts have been made in recent years. This is mainly due to the paucity of gravity anomaly data sets and their rather poor distribution in the region.

The importance contributions from the determination of the geoid can be summarised as follows:

- better understanding of the geology and the geophysics through geodetic perspective, since it is possible to study the gravity field features directly from the geoid. Variations in gravity field can be used in geothermal exploration and geophysical prospecting, especially in areas of potential oil and gas bearing structures (Hwang & Shih, 1998).
- greater use of GPS in height determination with high precision since it has the potential to replace the costly, labourious and time-consuming spirit levelling. It permits the determination of orthometric heights by combining geoid heights computed from a geoid model and ellipsoidal heights obtained from the GPS observations.
- more accurate connection of one local vertical datum to another, especially for the control of levelling networks (Kadir *et. al.*, 1999) and verification of apparent sea slope along the coasts (Ses & Gilliland, 1996).

In this section, the best available data is used to compute a preliminary national geoid and compare to a global gravity model. Comparisons are also made with the geoid produced involving some 230 GPS points that have precise levelling heights across the nation.



**Figure 6** Topographical features of Peninsular Malaysia

#### **2.2.1.1 Topographical Setting**

Peninsular Malaysia extends from latitude 1° 20' N to 6° 40' N and from longitude 99° 35' E to 104° 20' E. The total length of the country is roughly 804 km. Figure 6 illustrates the topographical features of Peninsular Malaysia. The Main Range of mountains trend roughly in a north-south direction with maximum elevations of over 2000 metres. It is long, narrow and rugged and lies nearer to the western side of the country. From the Thailand border, it runs continuously southwards into Negeri Sembilan where its height gradually diminishes and the hills merge with the coastal plain near the coast. The relief becomes less pronounced southwards especially over much of south of Pahang and Johor.

Narrow stretches of lowland lie on either side of the Main Range. On the eastern side of Peninsular Malaysia, another major barrier occurs. The barrier consists of several close parallel ranges with relatively deep valleys between. The ranges extend from the Kelantan coast in the north to the Pahang coast in the south. The heights in general are less than those in the Main Range.

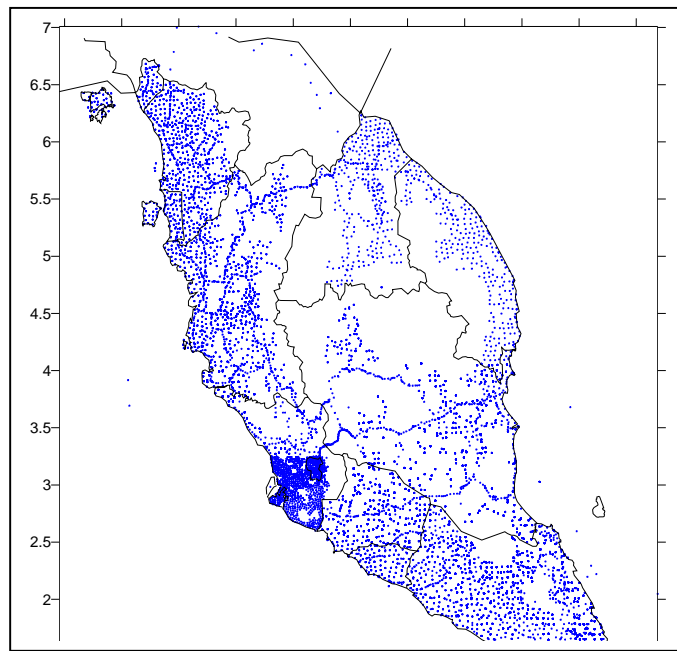
#### **2.2.1.2 Data Compilation and Preparation**

The different types of data used in the present study are as follows:

***The EGM96 Global Geopotential Model:*** Global geopotential models (GGMs) describe the Earth's gravitational potential in terms of an infinite series of spherical harmonics outside the Earth's attracting masses. They are determined by a combination of satellite and terrestrial observations and used as reference fields in the determination of local and regional geoids. The geopotential is usually given as a truncated set of harmonic coefficients. The maximum complete degree and order of these expansions has now reached 360 (Kirby & Featherstone, 1997), corresponding to a 111km wavelength at the equator. The smallest wavelength of geoid undulation that can be modeled is  $180^\circ/360$  or  $0.5^\circ$ , which is equivalent to about 55 km.

The most recent estimate of the global gravity field that is currently available is the Earth Gravity Model 1996 or EGM96. It is the national Aeronautics and Space Administration (NASA), National Imagery and Mapping Agency (NIMA), and The Ohio State University (OSU) joint global geopotential model in spherical harmonics complete to degree  $l$  and order  $m$  360 (Lemoine *et. al.*, 1997). It is also considered as the best global geopotential model currently available. This is due to the inclusion of new data, improved computational procedures and the use of geodetic satellites with different inclinations in to improve detection of low frequencies at low and mid latitudes. Globally, EGM96 is estimated to give the geoid undulation with an accuracy of 1 m.

**Terrestrial gravity data:** Generally, the gravity data over Peninsular Malaysia and the adjacent marine areas is poor. The data used in this study are 10400 point free air gravity anomalies which encompass the Peninsular Malaysia and surrounding marine area:  $0^{\circ} 00' N \leq \phi \leq 8^{\circ} 00' N$  and  $99^{\circ} 00' E \leq \lambda \leq 105^{\circ} 00' E$ . The data belong to the DSMM Gravity Data Bank and are referenced to the International Standardisation Network 1971 (IGSN71).



**Figure 7** The geographical distribution of the available gravity measurements

Figure 7 shows the geographical distribution of the available gravity measurements. The distribution varies from a very dense network in the Wilayah Persekutuan Kuala Lumpur, to less dense in the southern and western regions, to a sparse configuration in the central region and along the east coast.

**Digital Elevation Model (DEM):** DEM is a statistical representation of selected points on a continuous terrain relief in a digital form. It is used to better account for the terrain effects on the gravity field. The DEM for Peninsular Malaysia comes from the 30'' point topography database, GTOPO30 which is distributed by US Geological Survey.

According to Higgins, *et. al.* (1996), the aims of using DEMs in the computation of geoid are firstly to better account for the terrain effects on the gravity field by terrain related corrections. Secondly, it is used to interpolate the irregularly spaced raw gravity data.

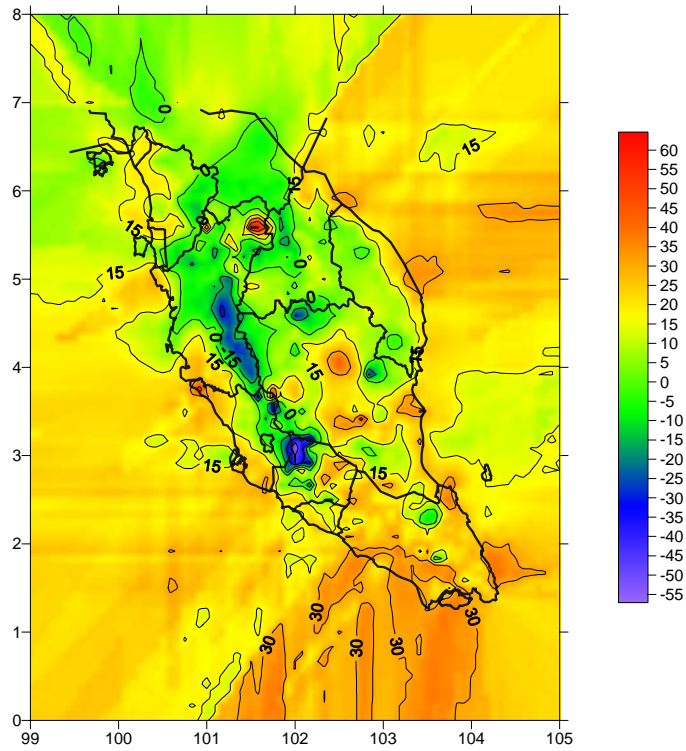
As mentioned, the gravity data over Peninsular Malaysia is generally poor. The middle area especially contains sparse or no gravity data at all. This necessitates some form of fill-in by prediction using DEM. Terrain corrections were computed from the DEM using the 1-D FFT technique. These corrections were then applied to the residual gravity anomalies (after the removal of the long wavelength contributions of EGM96). The resultant gravity data set thus obtained is referred to as terrain corrected data.

***Satellite altimetry-derived marine gravity data:*** The satellite altimetry-derived geoid heights data was extracted from a database of the Kort og Matrikelstyrelsen (KMS) or the Danish National Survey in Denmark. It comes on a 1.5' by 1.5' grid, covering both land and marine areas.

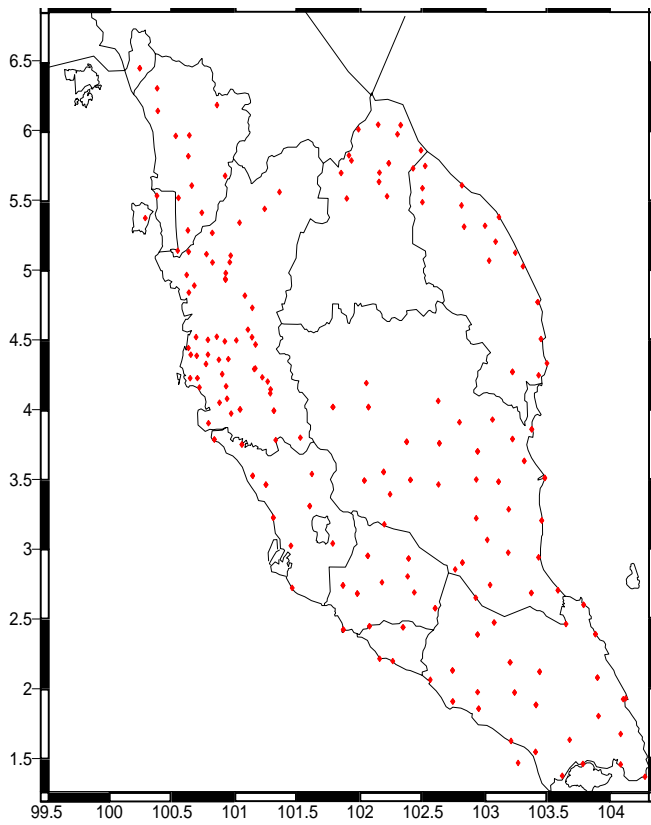
Using a small program that reads height data less than or equal to zero, the marine data were selected. A number of erroneous observations was deleted in accordance to the following adopted criteria:

- subsatellite points close to the shore line;
- short arcs;
- error value of more than 0.16 or 0.18; and
- high sea-surface heights.

From these selected altimeter data, the long wavelength effects of EGM96 were removed. Figure 8 illustrates a map of free air gravity anomalies contoured at 5 mGals for the marine and land areas around Peninsular Malaysia.



**Figure 8** A map of free-air gravity anomalies (contour interval is 15 mgals)



**Figure 9** The distribution of 230 GPS points having both the ellipsoidal and orthometric heights to derive  $N_{GPS}$  and be compared with  $N_{GRAV}$  and  $N_{EGM96}$

**GPS and spirit-levelled height data:** A total of 230 GPS points that have elevations established by precise levelling were compiled. Figure 9 illustrates the distribution of these points, which varies from a uniform network to very few points in the northern central region. The geometric geoid height at these points will be compared with the gravimetric geoid height derived from EGM96 and local model solutions. This will give some indication of the accuracy of the gravimetric geoid derived.

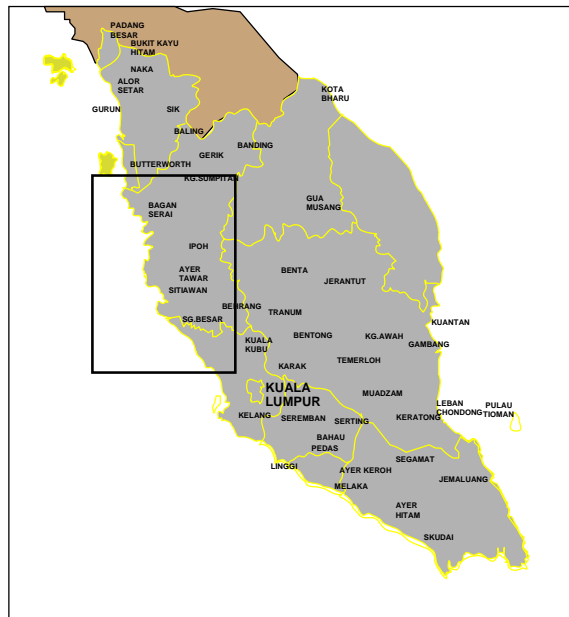
### **2.2.1.3 Geoid Computation Method**

As mentioned, the geoid can be defined as the gravity equipotential surface which best approximates mean sea level over the whole earth and as such it also acts as the datum for orthometric height system (Kuang *et. al.*, 1996). Since the geoid reflects the earth's gravity field, its shape is irregular and undulating. So, a reference surface of an ellipsoid, a regular mathematical figure, is selected to approximate the geoid surface. The geoidal height and the deflection of vertical describe the relationship between the two surfaces and they can be determined by various approaches. Many advances in the field of geoid computations, both in terms of theory and algorithm development have occurred in the past decade. These include the gravimetric method, the astro-geodetic method, satellite dynamic solution for potential coefficients and direct determination from 3-D geocentric coordinates and orthometric heights (Vanicek & Krakiwsky, 1986; Torge, 1980). Each of these approaches has its advantages and disadvantages.

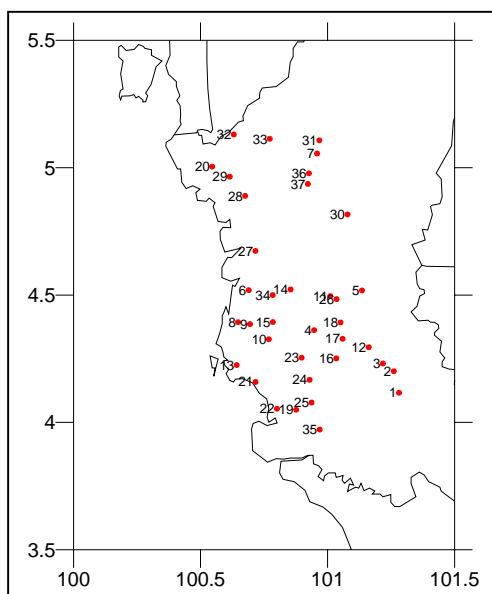
A gravimetric geoid model provides a very high resolution and local accuracies. However, one major drawback that it has is that it is subject to long-wavelength systematic effects. With the recent developments in methodology and techniques, older geoids based on astro-geodetic have now been superseded by global geoids evaluated using gravimetric methods. Nowadays GPS technique allows the determination of geoid heights  $N$  of points of which height is known from levelling  $H$ . It provides geocentric position of the point with  $h$  as the height above a reference ellipsoid such as WGS-84 or GRS80 by the relationship  $N = h - H$ .

### 2.2.2 Detailed Study Of Datum Bias In State of Perak

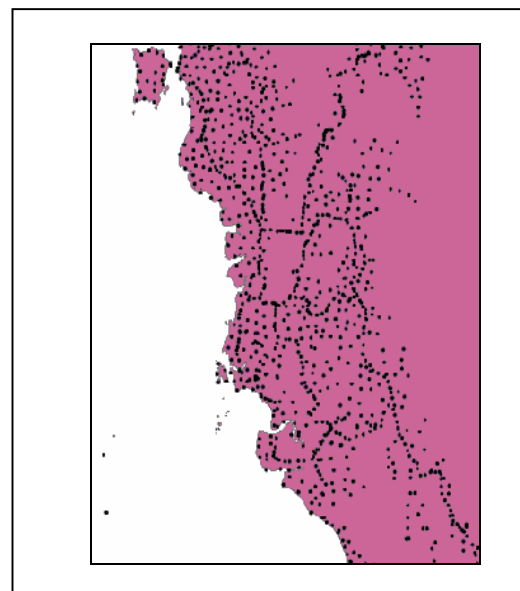
The selected test network consists of a subset of bench marks from the PLN in the northwest of Peninsular Malaysia. It covers the zone with limits  $3^{\circ} 30' \text{ N} \leq \phi \leq 5^{\circ} 30' \text{ N}$  and  $100^{\circ} 00' \text{ E} \leq \lambda \leq 101^{\circ} 30' \text{ E}$ . This translates to an approximate network



**Figure 10** Location of test area for GPS heighting study



**Figure 11** Distribution of GPS stations and GPS-occupied bench marks



**Figure 12** Distribution of gravity data



coverage area of roughly 220km x 160km as depicted in Figure 10. The nature of the terrain around the test area is relatively undulating. Height variation is from 2.15m to 88.60m along the GPS profile.

The chosen network consists of 57 stations as depicted in Figure 11, whilst Figure 12 shows the distribution of the gravity data over the area. The aim of selecting bench marks from the PLN was to provide data for constructing and evaluating contours of geoidal separation across the project area. The bench marks were so selected in order to give good point distribution through the network. The inter-station distance between the selected points were between 2 to 28 km with a mean spacing of 14 km. This also ensured inter-station drive times to be about 30 minutes since more than one session in a day was involved in the campaign.

#### **2.2.2.1 GPS on Bench Marks: Network and Computation**

A total number of 43 pre-selected bench marks and 14 GPS stations have been occupied by GPS. The area of study in the north of Peninsular Malaysia was chosen for this work. Specifically, this area covers most of the state of Perak.

The project utilised six Trimble 4000 SSE GPS receivers and Trimble Compact L1/L2 with ground plane antennas. The Trimble SSE is a dual-frequency, full wavelength GPS receiver. In order to achieve a relatively high degree of accuracy, most stations were occupied twice. As many as four receivers were moved at a time in a leapfrog fashion, re-observing two stations previously occupied. Each GPS loop was arranged so that it contained observations from at least two different sessions. The survey was designed in such a way so that there would be redundant information to detect data outliers.

There were a total of 22 sessions in the GPS campaign with two observation sessions per day. The antennae were orientated to the North throughout the campaign. GPS data was collected above a 15-degree elevation mask. The baselines were observed for a period of

between 2 to 3 hours at a 15-second epoch rate. Six or more satellites were continuously visible at all times. The survey was conducted during periods having Vertical Dilution of Precision (VDOP) less than four.

Primary network connections were also made to fourteen GPS stations of the Peninsular Malaysia Scientific and Geodetic Network (PMSGN) and GPS densification network. As a matter of providing initial quality control of the project, a preliminary daily baseline solution was performed each evening. Figure 13 shows the network configuration of the ‘GPS on BMs’ project.

Trimble’s proprietary processing package GPSurvey version 2.30 software was used to process the non-trivial vectors at a 30-second epoch and 20 degrees cutoff angle, whilst Geolab v2.4C was used in the network adjustment. Double difference carrier-phase observations were used in a least-squares adjustment to determine the fifty-six baselines. Three stations from the Peninsular Malaysia Scientific and Geodetic Network (PMSGN) were held fixed in the adjustment. Broadcast ephemerides were used in all reductions. This resulted in forty-three ellipsoidal heights referred to the WGS-84 ellipsoid. Table 1 gives a description of the occupied points involved in the campaign.

**Table 1** Network point type summary

Station Description	Number of Stations	Symbol	Remarks
Bench Mark	36	●	Intermediate Bench Mark
	7	○	Standard Bench Mark
GPS Points	3	△	Peninsular Malaysia Scientific and Geodetic Network (PMSGN)
	11	▲	GPS densification network

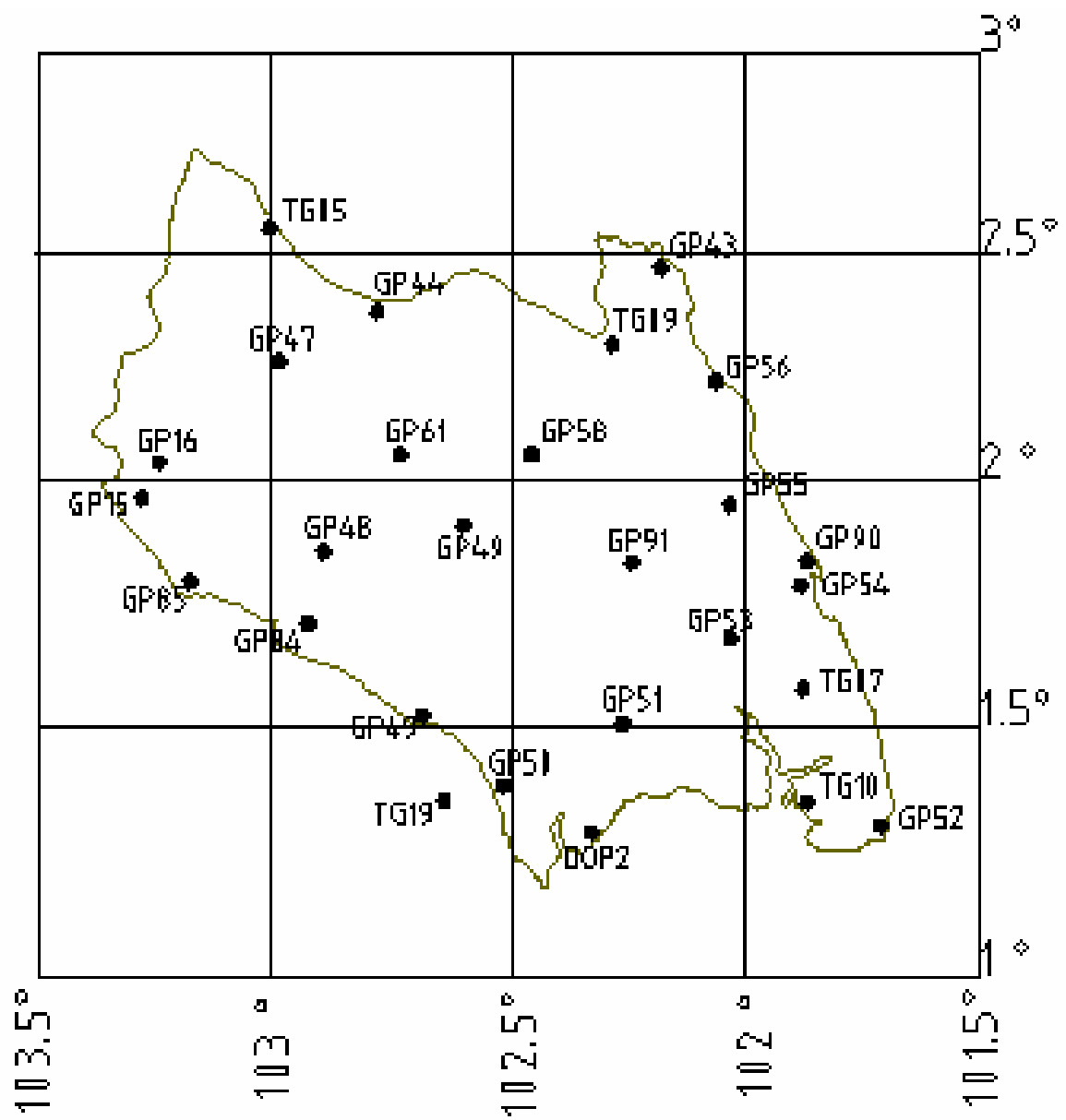


### 2.2.3 Investigation On Accuracy of GPS Heighting In The State of Johor

The Department of Survey and Mapping Malaysia kindly made available twenty-seven GPS derived co-ordinates in Johore State co-located with leveling for this study. The GPS derived co-ordinates on WGS84 datum associated with leveling tabulated in Table 2 were given in terms of geodetic latitude ( $\phi$ ), geodetic longitude ( $\lambda$ ), ellipsoidal height (h) and orthometric height (H). The distribution of this data set is shown in Fig. 14.

ST. NO	NAME	$\phi$	$\lambda$	h	H
1	DOP2	1.380	103.610	90.910	83.800
2	GP15	2.060	102.560	4.002	1.960
3	GP16	2.130	102.730	36.230	33.470
4	GP43	2.600	103.780	11.180	4.990
5	GP44	2.470	103.070	55.650	51.870
6	GP47	2.390	102.930	51.180	47.860
7	GP48	1.980	102.930	164.750	160.990
8	GP49	1.630	103.200	7.360	1.950
9	GP50	1.550	103.400	10.120	3.810
10	GP51	1.630	103.670	42.710	35.520
11	GP52	1.370	104.270	67.810	58.720
12	GP53	1.800	103.900	31.820	24.000
13	GP54	1.930	104.090	12.880	4.660
14	GP55	2.080	103.890	36.740	29.330
15	GP56	2.390	103.870	11.570	4.680
16	GP58	2.120	103.430	77.620	72.100
17	GP59	1.970	103.230	45.180	40.150
18	GP61	2.190	103.190	41.830	37.320
19	GP84	1.860	102.940	7.130	3.150
20	GP85	1.910	102.740	4.930	1.880
21	GP90	1.930	104.110	10.780	2.470
22	GP91	1.880	103.690	26.310	19.550
23	TG05	2.650	102.920	193.730	190.800
24	TG07	1.680	104.070	194.270	186.200
25	TG09	2.460	103.640	223.000	216.700
26	TG10	1.460	104.070	193.250	183.500
27	TG19	1.470	103.260	136.070	130.400

**Table 2** Johor GPS data associated with leveling.



**Fig. 14.** The distribution of the GPS data in Johor State

### 2.2.3.1 Computations

There are several approaches with which to transform GPS ellipsoidal heights to orthometric heights using either the point mode equation or the relative mode equation . The ellipsoidal height  $h$  derived from GPS measurements is given above a geocentric ellipsoid (which is WGS84 for GPS). Also the geoid undulations are required above the same ellipsoid to determine orthometric heights of interest. While ellipsoidal height  $h$  might be very accurate to the several millimeter level, the accuracy of the orthometric height, which is the height of interest is limited by the uncertainty of the geoid undulation  $N$  . Some of These methods used to evaluate  $N$  in this study are reviewed below.

#### 2.2.3.1.1 Gravimetric Geoid

A regional gravimetric geoid model over the area of South-East Asia was computed from  $1^\circ \times 1^\circ$ ,  $30' \times 30'$  mean surface gravity anomaly data combined with a truncated EGM96 (70,70) spherical harmonics potential coefficient set ( Kadir *et al.*,1998 ). To compute the gravimetric geoid in South-East Asia, the Molodensky modified Stokes's kernel was used instead of the original ellipsoidal Stokes's kernel. This reduces truncation errors as the former tapers off more rapidly than the latter (the influence of distant gravity anomalies on local geoid heights is, thus reduced). The reduction is proportional to the degree  $L$  of the satellite model being used to recover the long wavelength component of the geoid. The total gravimetric geoid was obtained from the summation of the long wavelength and the short wavelength components. This solution was referred to the World Geodetic System 1984 (WGS-84). The gravimetric geoid heights ( $N_{GRAV}$ ) at the twenty-seven GPS stations in Johore State were estimated from the above geoid model.

#### 2.2.3.1.2 Geopotential Geoid

One of the methods to estimate global geoid undulations is by using geopotential models. These are mathematical models in which gravitational potential of the Earth can be

expanded into a series of spherical harmonics. In this study, the EGM96 geoid heights ( $N_{EGM}$ ) and the OSU91A geoid heights ( $N_{OSU}$ ) on GRS80 were estimated from EGM96 and OSU91A model coefficient sets together with the parameters of the GRS80 ellipsoid.

### 2.2.3.1.3 Geometric Geoid

The geometric geoid could be derived from a combination of GPS measurements associated with spirit leveling. The end product of GPS measurements are either the ellipsoidal co-ordinates ( $\phi, \lambda, h$ ) or Cartesian co-ordinates ( $X, Y, Z$ ) on the WGS84 system. The spirit leveling results in heights referenced to the geoid. These heights are termed orthometric heights ( $H$ ). The geoid height ( $N$ ) at any control station with ellipsoidal co-ordinates ( $\phi, \lambda, h$ ) and known orthometric height ( $H$ ) is calculated in a point mode using equation (22) or in a relative mode from equation (23)

$$N_P = h_P - H_P \quad (25)$$

$$N_B = N_A + \Delta h_{AB} - \Delta H_{AB} \quad (26)$$

If the co-ordinates of the control station were given in rectangular form ( $X, Y, Z$ ) then they have to be converted into ellipsoidal form to deduce the ellipsoidal height ( $h$ ) before applying the above equations for geoid height recovery. The ellipsoidal co-ordinates ( $\phi, \lambda, h$ ) could be obtained from the Cartesian co-ordinates ( $X, Y, Z$ ) associated with the parameters of the WGS84 datum using the non-iterative reverse conversion formulae given in Bowring (1985) as:

$$\phi = \arctan \left[ \frac{Z + \varepsilon^2 b \sin^3 u}{P - \varepsilon^2 a \cos^3 u} \right] \quad (27)$$

$$\lambda = \arctan \left[ \frac{Y}{X} \right] \quad (28)$$

$$h = P \sec \phi - v \quad (29)$$

with:

$$u = \arctan \left[ \frac{aZ}{bP} \right]$$

$$P = \sqrt{X^2 + Y^2}$$

$$\varepsilon = \frac{e^2}{1 - e^2}$$

where;

- u : the parametric latitude;
- a : the semi-major axis of the reference ellipsoid;
- b : the semi-minor axis of the reference ellipsoid;
- e : the first eccentricity of the reference ellipsoid.
- $\varepsilon$  : the second eccentricity of the reference ellipsoid.

Using the data of Table 2, the geometric geoid ( $N_{GPS}$ ) on GRS80 ellipsoid at the twenty-seven GPS stations is computed from equation (25).

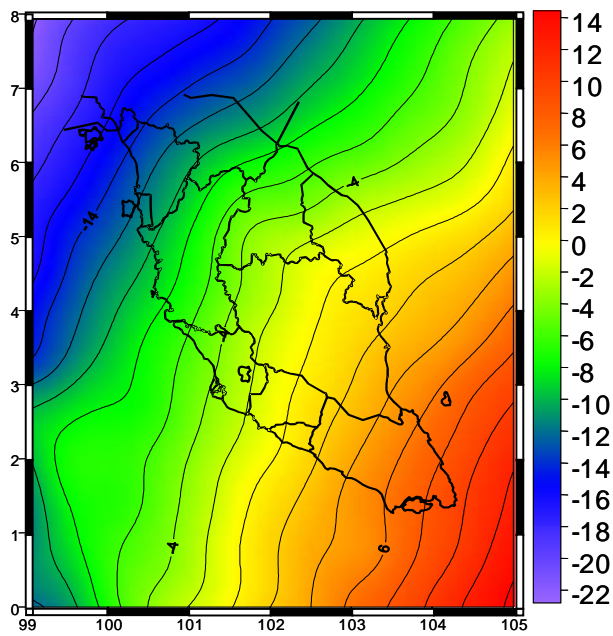


## **3.0 RESULTS**

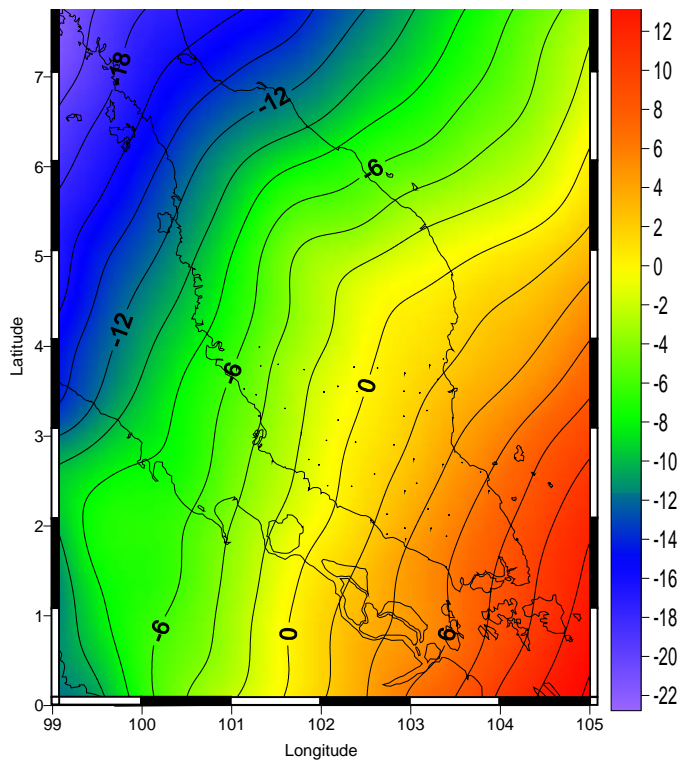
### **3.1 Results of The Vertical Datum Bias For Peninsular Malaysia and Comparisons with GPS/Levelling Heights**

The result from the geoid computation takes the form usually indicated by a contour plot whereby it shows the position of the geoid with respect to the selected ellipsoid. Figures 15 and 16 illustrate the contours of the EGM96 and preliminary national geoid model for Peninsular Malaysia (referred to here forth as MYGeoid02) respectively. It should be noted that MYGeoid02 is considered preliminary because of the lack of gravity data in the central region as mentioned before. However, based on the dense coverage of the gravity points, it can be considered as being more reliable in the western coastal regions and in the southern part of Peninsular Malaysia.

Both geoids, EGM96 and MYGeoid02, are rising from north-west to south-east. They coincide with the reference ellipsoid GRS80 roughly in the central region, from 102°E meridian to 103.5°E meridian and from 2°N to 5°N. The contours descend from 10m in the south to -15m in the north. Results also indicate that the maximum and minimum values of the MYGeoid02 are 9.45m and -14.66m respectively. Whilst for the EGM96 geoid, the figures are 9.53m and -14.47m. MYGeoid02 and EGM96 have a mean undulation value of 9.45m and 9.53m respectively.



**Figure 15** EGM96 geoid model for Peninsular Malaysia at 5-min grid (contour interval is 2m)



**Figure 16** Preliminary geoid model for Peninsular Malaysia, MYGeoid02 at 2-min grid (contour interval is 2m)

GPS/Levelling geoid heights for 230 points in Peninsular Malaysia are also computed by subtracting adjusted orthometric heights from the corresponding GPS ellipsoidal heights. Comparisons are made between the offset of EGM96 and MYGeoid02 models with these GPS-derived geoid height at each of those points. The available N- values from GPS are compared to get a qualitative estimate of the fit of the geoids, GPS and the orthometric heights. Table 3 shows descriptive statistics of the absolute fit of MYGeoid02 and EGM96 to the 230 GPS/Levelling control data. The removal of suspected data outliers (2.6%) in the geometric geoid improved the standard deviations considerably as shown in Table 4.

**Table 3.** Comparison of geoid models to GPS-derived geoid heights before removal of suspected data outliers (values in metres)

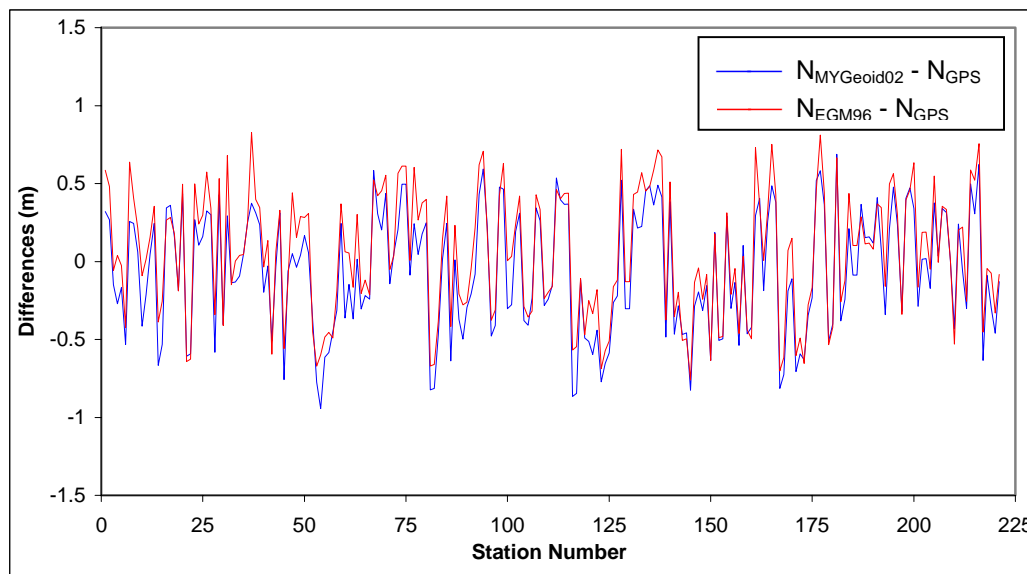
<b>Geoid Model</b>	<b><math>N_{\text{Model}} - N_{\text{GPS}}</math></b>			
	<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std. Dev.</b>
MYGeoid02	-2.653	1.785	0.100	0.471
EGM96	-2.710	1.687	-0.051	0.498

**Table 4.** Comparison of geoid models to GPS-derived geoid heights after removal of suspected data outliers (values in metres)

<b>Geoid Model</b>	<b><math>N_{\text{Model}} - N_{\text{GPS}}</math></b>			
	<b>Minimum</b>	<b>Maximum</b>	<b>Mean</b>	<b>Std. Dev.</b>
MYGeoid02	-0.943	0.688	-0.065	0.389
EGM96	-0.760	0.827	0.052	0.402

The two geoid models are found to have approximately the same relative accuracy. Results show that the standard deviations of the differences are  $\pm 40$  cm for the EGM96 solution and  $\pm 39$  cm for MYGeoid02 solution. The 40 cm magnitude of the fit can be considered as satisfactory. From the inspection of the standard deviations of both geoids, it can be seen that MYGeoid02 represents only a slight improvement over EGM96. Some possible explanations for this include:

- increased resolution since MYGeoid02 is computed on a 1-min grid compared to the 5-min resolution of EGM96;
- additional new gravity data; and
- use of DEM data since MYGeoid02 features in the central mountain range of Peninsular Malaysia strongly correlates with the terrain.

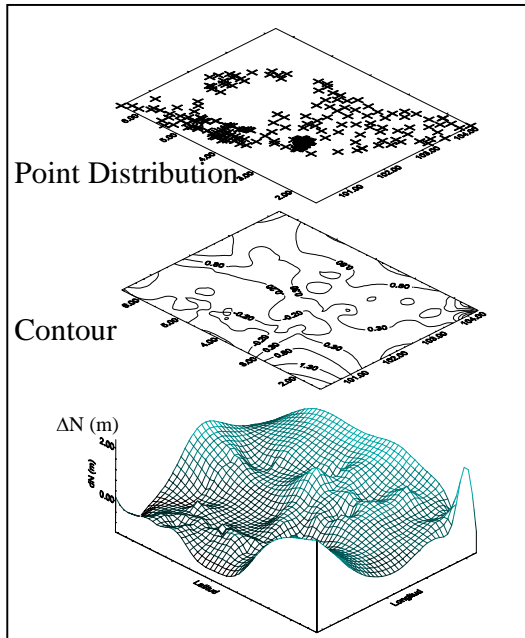


**Figure 17** Comparison between gravimetric geoids with geometric geoid

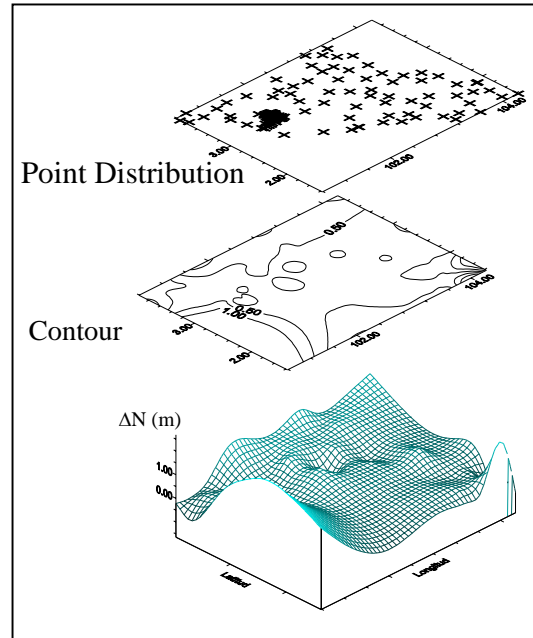
Figure 17 illustrates the results obtained from the comparison between the two gravimetric geoids and the geometric geoid. From the examination of the Figure, it may

be noted that both EGM96 and MYGeoid02 geoids exhibit similar trends. Overall, it can be concluded that, with the limited amount of gravity points available in Peninsular Malaysia, the use of DEM, the inclusion of satellite-altimeter-derived gravity data and the adoption of suitable computational techniques, MYGeoid02 is the more suitable geoid model for the transformation of GPS-derived ellipsoidal height differences to orthometric height differences.

Figure 18 and Figure 19 show the vertical datum bias for Peninsular Malaysia.



**Figure 18 :**  
Vertical Datum Bias For EGM'96



**Figure 19 :**  
Vertical Datum Bias For Gravimetric Geoid

Leveling and GPS data have been used to compute one flat surface using the equations as follows:

$$\delta N (\phi, \lambda) = a \Delta \phi + b \Delta \lambda \cos(\phi) + c$$

where :

- b = East-west direction tilt.
- a = North-south direction tilt.

$c$	= distance between two geoid models.	
$Tilt$	= maximum surface tilt	$= (a^2 + b^2)^{1/2}$
$Azimuth$	= tilt direction	$= \tan^{-1}(b/a)$ .

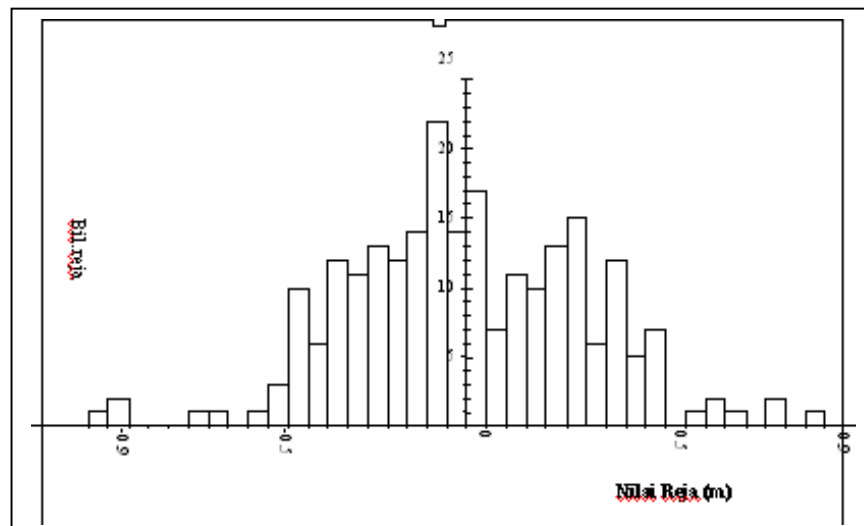
The variables  $a$ ,  $b$  and  $c$  are solved using least square adjustment. The result is shown in the following:

	<b>EGM96</b>	<b>Gravimetrik</b>
Number of data used	233	135
$a$ (sec)	0.1642	0.4529
$b$ (sec)	0.6085	0.2161
$c$ (m)	0.1142	0.1759
Tilt (sec)	0.6303	0.5018
Ppm	3.1	2.4
Azimuth (deg)	331.4929	21.6530

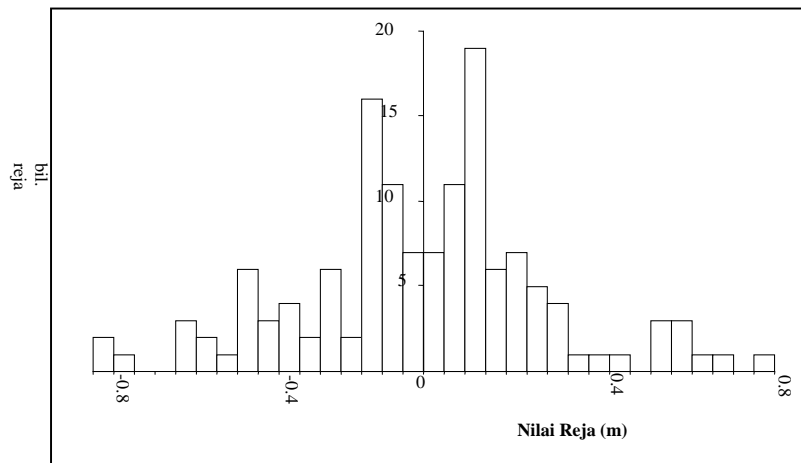
**Table 5:** Datum Bias Parameter

For Local gravimetric model, the area consists of latitude  $1.45^\circ \text{ N} - 3.5^\circ \text{ N}$  and longitude  $100.5^\circ \text{ E} - 104.5^\circ \text{ E}$ .

To ensure that the data fit the formed model, the residual must be small and has a normal distribution. This residual show the random error of data. In this study, the resulting residual shows that there is still blunder where in fig. 20 and 21, the residual close to 1 m. However, they still show the characteristic of a normal distribution.

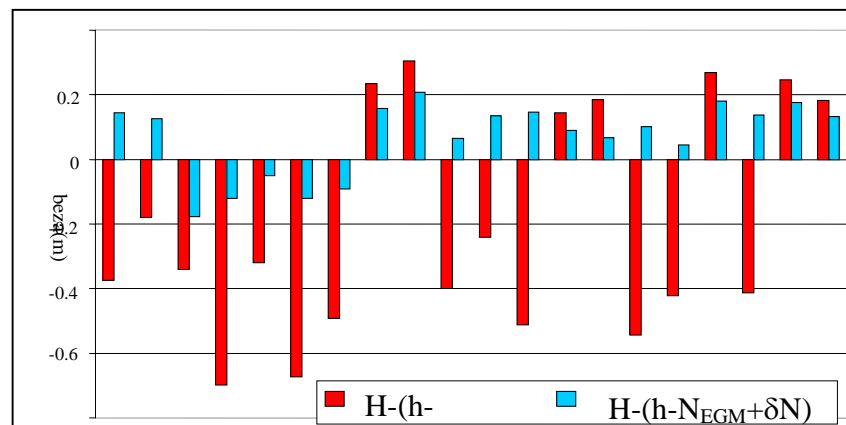


**Figure 20 :** Residual Histogram From Model Geoid EGM96 and  $N_{MSL}$ .



**Figure 21 :** Residual Histogram From Model Geoid Local Gravimetric and  $N_{MSL}$

Using the model explained above, the GPS heighting is conducted to obtain the orthometric height. The orthometric heights obtained here are close to the Mean Sea Level Height . Figure 22 show the result :



**Figure 22 :**Bar Chart Showing The Comparison Between The Orthometric Height With or Without  $\delta N$  ( EGM96 )

### 3.2 Results of The Detailed Study of Datum Bias In The State of Perak

Since GPS ellipsoidal heights and orthometric heights are available for the forty-three bench marks in the test network, a product of GPS-derived geoid heights was obtained. As the objective of this investigation is to confirm the potential of GPS approach in determining orthometric heights, a set of geoidal heights was also compiled from a local geoid. The extent of agreement to which GPS-derived geoid heights match gravimetric geoidal heights is used as an indicator of the ability of GPS to replicate spirit levelling.

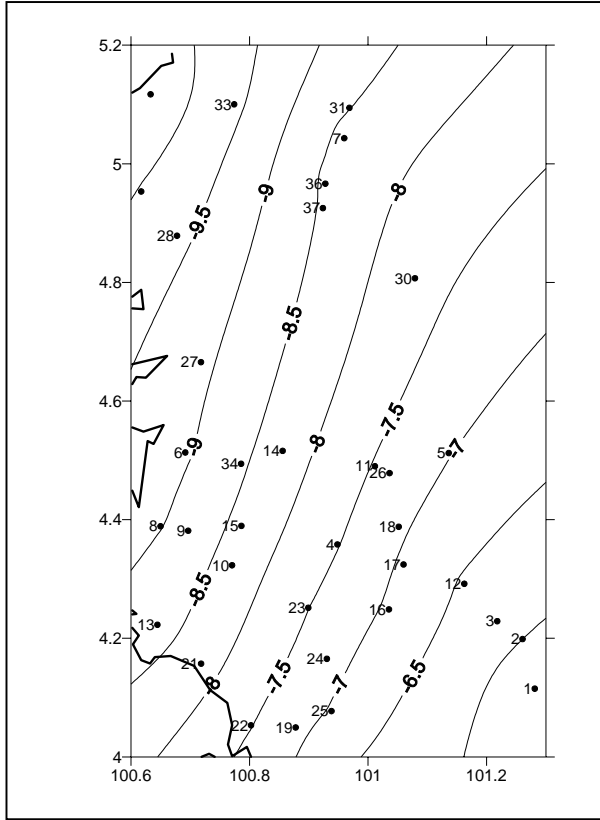
The analysis of the local geoid model to geoid heights determined from the ‘GPS on BMs’ project in the test area involves only thirty-seven bench marks; the remaining five bench marks are used as a check. The analysis is carried in two parts (Veronneau, 1993). The first part of analysis is to allow an investigation be made on the overall accuracy of the model in the test area. It involves the estimation of geoid heights at every station using the local geoid model (Figure 23). These geoid heights are interpolated using WINTER (Windows Interpolation) software which can be downloaded without charge from the AUSLIG website (<http://www.auslig.gov.au/geodesy/ausgeoid/geoid.htm>). The software can interpolate in a batch mode a file of positions using either bilinear or bicubic interpolation. This is followed by a direct comparison to the respective geoid heights as derived from GPS observations combined with levelling data using the following expression:

$$\delta N_I = (h_{GPS} - H_{Levelling}) - N_{Model}$$

where  $h_{GPS}$  is the GPS derived ellipsoidal height,  $H_{Levelling}$  is the orthometric height and  $N_{Model}$  is the computed geoidal height.

The results of the comparison is summarised in Table 5. The standard deviation of the discrepancies of the GPS-derived geoid heights from the local geoid model on thirty-seven bench marks is 15 cm. The value of  $\delta N_I$  ranges from 11.8cm to 51.2cm.





**Figure 23** Local geoid model for the State of Perak

The second part of the analysis concerns with the evaluation of the accuracy in relation to the baseline distance between stations. The computed geoidal height differences,  $\Delta N_{Model}$  along forty baselines in the network are compared with the corresponding results obtained from GPS derived ellipsoidal height differences,  $\Delta h_{GPS}$  and orthometric height differences,  $\Delta H_{Levelling}$ . The comparison can be expressed as:

$$\delta N_2 = (\Delta h_{GPS} - \Delta H_{Levelling}) - \Delta N_{Model}$$

This type of analysis is especially useful in identifying problematic stations when associated repeatedly to baselines having largest errors in ppm. The results of the comparison are shown in Table 6. The value of  $\delta N_2$  ranges from -35.3cm to 28.7cm. It can be seen that larger discrepancies are obtained when the baselines cross the geoid contour lines (Figure 23) and made the geoid height more significant.

**Table 6** Discrepancies between the local geoid model and geoid heights derived from GPS observations for 37 bench marks in Perak

Station	Longitude $\lambda$ (°)	Latitude $\phi$ (°)	$h_{\text{GPS}} - H_{\text{Levelling}}$ (m)	$N_{\text{Model}}$ (m)	$\delta N$ (m)
A0085	101.2814	4.11628	-5.696	-5.578	-0.118
A0089	101.2607	4.20096	-5.940	-5.968	0.028
A0092	101.2179	4.23137	-6.224	-6.171	-0.053
A0123	100.9473	4.36234	-7.535	-7.712	0.177
A0152	101.1357	4.51839	-7.035	-7.014	-0.021
A0363	100.6897	4.51937	-9.078	-9.067	-0.011
A0424	100.9588	5.05571	-8.406	-8.622	0.216
A0500	100.6478	4.39344	-9.034	-9.058	0.024
A0585	100.6946	4.38580	-8.832	-8.868	0.036
A0600	100.7688	4.32683	-8.316	-8.564	0.248
A0635	101.0108	4.49590	-7.598	-7.702	0.104
A0701	101.1621	4.29531	-6.449	-6.643	0.194
A0726	100.6425	4.22529	-8.712	-8.880	0.168
A0832	100.8545	4.52206	-8.176	-8.382	0.206
A0840	100.7847	4.39410	-8.382	-8.516	0.134
A0974	101.0345	4.25155	-6.942	-7.177	0.235
A0979	101.0592	4.32813	-6.940	-7.192	0.252
A0983	101.0511	4.39260	-7.093	-7.268	0.175
A1285	100.8766	4.05006	-7.128	-7.533	0.405
A1381	100.5459	5.00448	-10.293	-10.647	0.354
A1396	100.7165	4.15890	-8.202	-8.370	0.168
A1555	100.8010	4.05374	-7.527	-7.890	0.363
A1597	100.8980	4.25431	-7.517	-7.867	0.350
A1601	100.9294	4.16724	-7.160	-7.572	0.412
A1606	100.9372	4.07804	-6.968	-7.333	0.365
A1622	101.0355	4.48418	-7.349	-7.504	0.155
A1802	100.7162	4.67351	-9.112	-9.151	0.039
A1831	100.6757	4.88922	-9.656	-9.827	0.171
A1839	100.6150	4.96478	-10.003	-10.124	0.121
S0091	101.0785	4.81665	-7.715	-7.633	-0.082
S0294	100.9676	5.10751	-8.559	-8.704	0.145
S0376	100.6309	5.13059	-10.356	-10.482	0.126
S0379	100.7725	5.11354	-9.644	-9.723	0.079
S0411	100.7842	4.50004	-8.579	-8.649	0.070
A0933	100.9695	3.97195	-6.478	-6.990	0.512
S0461	100.9268	4.97805	-8.437	-8.633	0.196
S0462	100.9227	4.93639	-8.462	-8.586	0.124
No. of points			37		
Minimum ( $\delta N_1$ )			-0.118 m		
Maximum ( $\delta N_1$ )			0.512 m		
Mean ( $\delta N_1$ )			0.164 m		
Std. Dev. ( $\delta N_1$ )			$\pm 0.146$ m		
RMS ( $\delta N_1$ )			$\pm 0.218$ m		

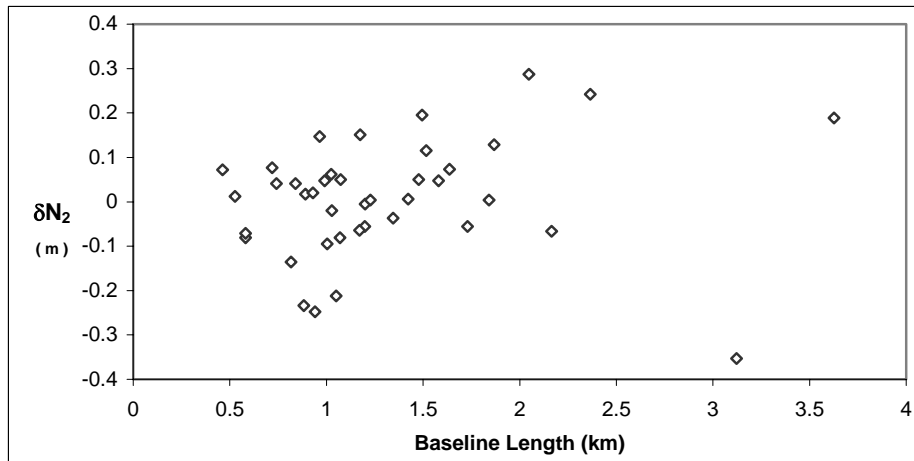
**Table 7** Comparison of geoidal height differences, i.e. observed ( $\Delta h - \Delta H$ ) and computed  $\Delta N$

	From	To	Length, $s_i$ (m)	$\Delta h_{GPS} - \Delta H_{Levelling}$ (m)	$\Delta N_{Model}$ (m)	$\delta N_2$ (m)
	A0933	A1285	13456.596	0.506	0.543	-0.037
	A0933	A1606	12268.216	0.347	0.343	0.004
	A1285	A1606	7407.234	-0.159	-0.200	0.041
	A1285	A1555	8393.492	0.398	0.357	0.041
	A1555	A1396	14943.816	0.675	0.480	0.195
	A1601	A1396	23652.946	1.040	0.798	0.242
	A1601	A1285	14222.600	-0.033	-0.039	0.006
	A1601	A1606	9900.655	-0.192	-0.239	0.047
	A1601	A1597	10239.186	0.357	0.295	0.062
	A1597	A0974	15160.135	-0.575	-0.690	0.115
	A0974	A0092	20478.826	-0.719	-1.006	0.287
	A0092	A0089	5819.130	-0.284	-0.203	-0.081
	A0089	A0085	9643.597	-0.243	-0.390	0.147
	A0085	A0974	31227.298	1.246	1.599	-0.353
	A0092	A0701	9403.042	0.224	0.472	-0.248
	A0701	A0979	11978.114	0.493	0.549	-0.056
	A0979	A0974	8900.701	0.002	-0.015	0.017
	A0979	A0983	7186.502	0.153	0.076	0.077
	A0983	A0123	11995.684	0.439	0.444	-0.005
	A0123	A0635	16364.921	0.063	-0.010	0.073
	A1622	A0983	10273.909	-0.256	-0.236	-0.020
	A1622	A0152	11743.539	-0.318	-0.469	0.151
	A0635	S0091	36257.152	0.120	-0.069	0.189
	A0726	A0585	18668.682	0.117	-0.012	0.129
	A0585	A0600	10502.000	-0.516	-0.304	-0.212
	A0585	A0500	5269.987	0.202	0.190	0.012
	A0585	A0840	10040.993	-0.447	-0.352	-0.095
	A0585	A0363	14779.518	0.249	0.199	0.050
	A0363	S0411	10704.687	-0.499	-0.418	-0.081
	S0411	A0840	11714.748	-0.197	-0.133	-0.064
	S0411	A0832	8172.608	-0.403	-0.267	-0.136
	A0363	A1802	17297.079	0.028	0.084	-0.056
	A1831	A1839	10727.120	0.347	0.297	0.050
	A1839	A1381	8834.709	0.289	0.523	-0.234
	A1839	S0376	18420.107	0.362	0.358	0.004
	S0376	S0379	15807.900	-0.712	-0.759	0.047
	S0379	S0294	21648.778	-1.085	-1.019	-0.066
	S0294	A0424	5811.602	-0.153	-0.082	-0.071
	A0424	S0461	9289.581	0.031	0.011	0.020
	S0461	S0462	4629.641	0.025	-0.047	0.072
No. of baselines, n Min. and Max. ( $\Delta h - \Delta H$ ) Min. and Max. $\Delta N$ Average length, s RMS RMS/s Mean relative accuracy, $\bar{a}$				40 -1.085 m, 1.246 m -1.019 m, 1.599 m 13080.926 m $\pm 0.129$ m 9.9 ppm 8.0 ppm		

Statistical analyses of the fitness of the two data sets are also carried out. First, the RMS value as defined as follows:

$$RMS = \pm \sqrt{\frac{\sum_{i=1}^n (\Delta h_{GPS} - \Delta H_{Levelling} - \Delta N_{Model})_i^2}{n}}$$

where  $n$  is the number of baselines and  $\Delta N$  value was interpolated from the respective geoids. The second statistic computed is the RMS/s where  $s$  is the mean length of all baselines.



**Figure 24** The values of  $\delta N_2$  as a function of baseline length

The values of  $\delta N_2$  from Table 7 are also plotted against the length of the baselines as shown in Figure 24. It demonstrates that the accuracy of the baseline degrades as the baseline length increases. Fotopoulos *et al.*, (2002) have conducted several tests using various baseline lengths. It was also found that the longer the length, the poorer the achievable  $\Delta H$  accuracy from GPS/geoid levelling. Among the reasons given for this result are partly due to the spatial decorrelation of GPS errors and the contribution of the

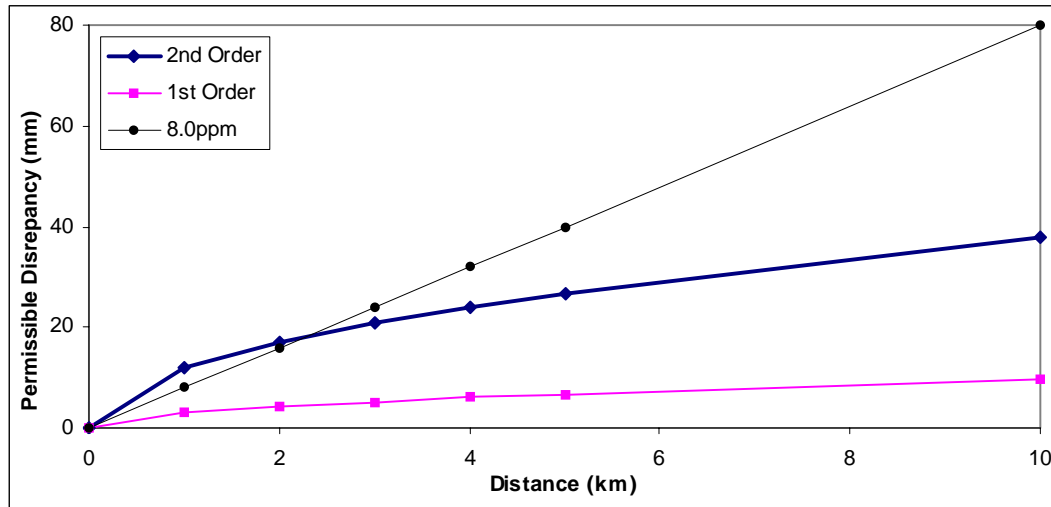
corrector surface model error.

The third value is the mean relative accuracy  $\bar{a}$  or the mean ppm value,

$$\bar{a} = \frac{1}{n} \sum_{i=1}^n \left| \frac{\Delta h_{GPS} - \Delta H_{Levelling} - \Delta N_{Model}}{s_i} \right|$$

where  $s_i$  is the individual baseline length. The results from the three statistical tests are also given in Table 7. It is shown that the RMS of the discrepancies between the the computed geoidal heights and the GPS-derived geoidal heights was 8.0 to 9.9 ppm of the distance separating the points.

The permissible discrepancy for leveling in Peninsular Malaysia is  $3\text{mm}\sqrt{\text{km}}$  for 1<sup>st</sup> order 2-way levelling and  $12\text{mm}\sqrt{\text{km}}$  for 2<sup>nd</sup> order levelling. Thus, the accuracy of spirit levelling is a function of the square root of the levelling distance. On the other hand, GPS accuracy depends on the interdistance between points. This is clearly demonstrated in Figure 9.15 whereby the 8.0 ppm line is plotted together with the levelling permissible discrepancy curves. The results demonstrate that the accuracy achievable by GPS heighting do not meet the requirements of the first order spirit levelling in this part of the country. It appears that at best GPS heighting technique can provide 2<sup>nd</sup> order levelling standard only for distances of up to 2.25km, at a precision that equates to 18mm.



**Figure 25** Comparison of accuracies of orthometric height differences from GPS (8.0ppm) and from spirit levelling

The accuracy and density of gravity data is paramount in any gravimetric determination. The test area is located close to the coast. This means that the altimeter data are affected by wave action and also by loss of signal lock through back scattering from the coastal land. To the north and south-east of the test area, there are disparities of the coverage of gravity data. These are probably among the reasons for the relatively less satisfactory performance of the gravimetric geoid in the test area.

Checks were also carried out with points not included in the analyses. Table 8 presents the results at five check points within the test area. The average difference between GPS-derived and local geoid heights is  $-0.0412\text{m}$ , with values ranging from  $-0.224\text{m}$  to  $0.079\text{m}$ . The standard deviation of the differences is  $0.1148\text{m}$ .

In practice, the accuracy of the relative orthometric height  $\Delta H$  is dependent on the accuracy of the geoidal height,  $\Delta N$ , and of the relative ellipsoidal height,  $\Delta h$ , in the relation:

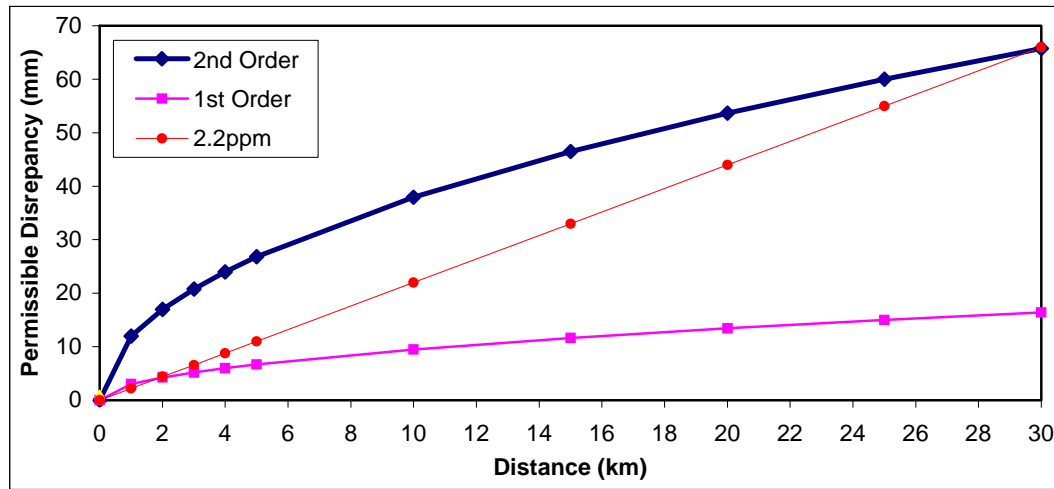
$$\sigma_{\Delta H}^2 = \sigma_{\Delta N}^2 + \sigma_{\Delta h}^2$$

**Table 8** Geoidal height differences of 5 check points

No.	Longitude $\lambda$ (°)	Latitude $\phi$ (°)	$N_{GPS}$ (m)	$N_{Model}$ (m)	Difference (m)
1	101.0108	4.49590	-7.598	-7.374	-0.224
2	101.2607	4.20096	-5.940	-5.923	-0.017
3	100.8545	4.52206	-8.176	-8.255	0.079
4	100.9294	4.16724	-7.160	-7.180	0.020
5	100.7162	4.67351	-9.112	-9.048	-0.064
Mean					-0.0412 m
Standard Deviation					$\pm 0.1148$ m

If the relative accuracy of  $\Delta h$  is taken as 2ppm over a line of 20km, the  $\sigma_{\Delta h}$  inferred is 40mm. Similarly, if  $\sigma_{\Delta N}$  is 15mm as a result of a ‘hypothetical’ improvement to MYGeoid02, the relative accuracy required for  $H$  is roughly 43mm. This equates to 2.2ppm.

Figure 23 shows a plot of the 2.2ppm line with the 1<sup>st</sup> and 2<sup>nd</sup> order levelling permissible discrepancy curves. It can be suggested that, with cm-accuracy geoid, the use of GPS in height surveys is comparable to 1<sup>st</sup> order levelling in not more than 2km. On the other hand, the technique exceeds 2<sup>nd</sup> order levelling for distances up to 30km.

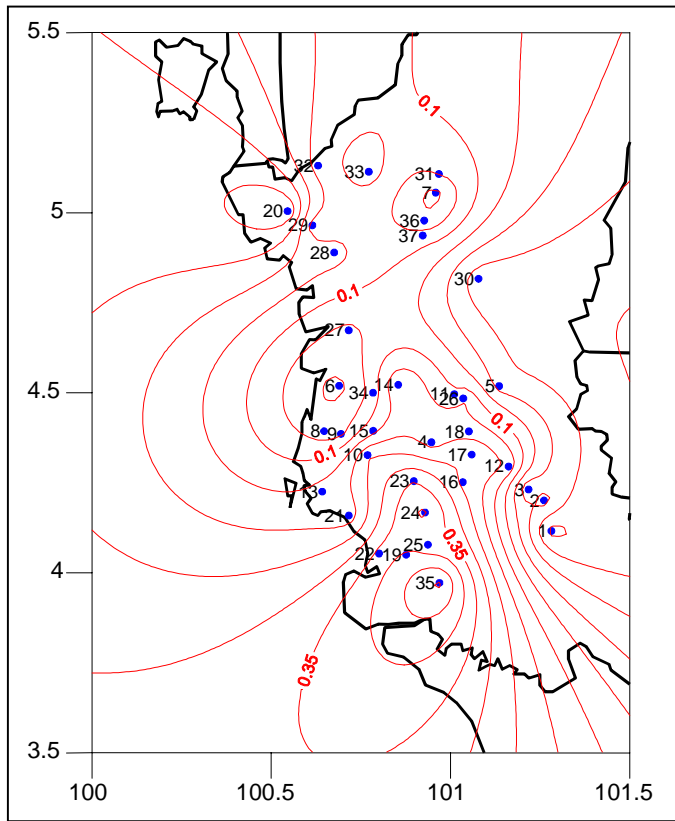


**Figure 26** Comparison of accuracies of orthometric height differences from GPS (2.2ppm) and from spirit levelling

### 3.2.1 Modelling Vertical Datum Bias

As can be seen from Table 6 the direct comparison between the local geoidal heights  $N$  and height differences between GPS heights ( $h$ ) and levelling heights ( $H$ ) of some 37 points results in discrepancies as denoted by  $\delta N_I$ . Figure 27 presents a contour plot of  $\delta N_I$  values, together with the data points which were used. The influence of the Main Range mountain chain can be clearly seen in the southern part of the test area where the largest values of  $\delta N_I$  are found. This is expected, since the topography features in the area are undulating and rough. This made both geoid modelling and spirit levelling less accurate than in flat areas.





**Figure 27** Contour Plot of Discrepancies ( $\delta N_l$ ) between the local geoid model and geoid heights derived from GPS observations at 37 common points of GPS and levelling in Perak (shown as solid circles; contour interval is 0.05m)

There are a number of factors that contribute to the computed values of  $\delta N_l$ . These values, as in Table 6, reflect the vertical datum inconsistencies between the available height data, long wavelength geoid errors, and GPS and levelling errors included in the ellipsoidal and orthometric heights (Vergos & Sideris, 2002). In Peninsular Malaysia, biases are known to exist in the computed local geoids, with values reported as -8cm, -41cm and -54cm (Fashir *et al.* 2000a; Fashir *et al.* 2000b; Kadir *et al.*, 2000). These inconsistencies could be improved by fitting a bias and tilt to the discrepancies. In order to reduce the amount of the deviations to a minimum, a three-parameter model or a planar surface function is applied (modified from Fashir *et al.* 2000a) with pointwise derivation given in Section 5.9 of Heiskanen & Moritz (1967):

$$\delta N = (h - H) - N = \sum_{i=0}^n \sum_{j=0}^n a_{ij} x^i y^j + v$$

or

$$\delta N = a_{00} + a_{01}y + a_{10}x + v$$

where  $a_{00}$  is the shift parameter between the vertical datum implied by the GPS/levelling data and the gravimetric datum,  $a_{01}$  and  $a_{10}$  are the north-south and east-west tilts of the plane surface,  $(i, j)$  denotes the horizontal location of the points,  $x$  and  $y$  are the horizontal coordinate components of the regression surface and  $v_i$  are the residuals to be minimised.

A four-parameter transformation model with one additional parameter added is represented by the following formula:

$$\delta N = a_{00} + a_{01}y + a_{10}x + a_{11}xy + v$$

where now  $a_{01}$ ,  $a_{10}$ , and  $a_{11}$  are the translation parameters in x, y and z axes between the coordinate system implied by the GPS data and the one implied by the gravimetric data.

It should be noted that the choice of the parametric form of the corrector surface model, which can vary from a simple plane to more complicated higher-order polynomial and trigonometric corrector surfaces, is not a trivial task (Kotsakis *et al.*, 2001; Featherstone, 2000). For the test carried out in the present study, a four-transformation model is used. The polynomial coefficients  $a_{00}$ ,  $a_{01}$ ,  $a_{10}$ , and  $a_{11}$  in the four-parameter transformation model can be estimated through a least squares fit and tested. Once these have been solved for, the gravimetric geoid can be corrected into a geoid compatible with the GPS-derived geoid. Thus, the resulting geoid will be associated with a local vertical datum, similar to the levelled heights given for the GPS-occupied stations.

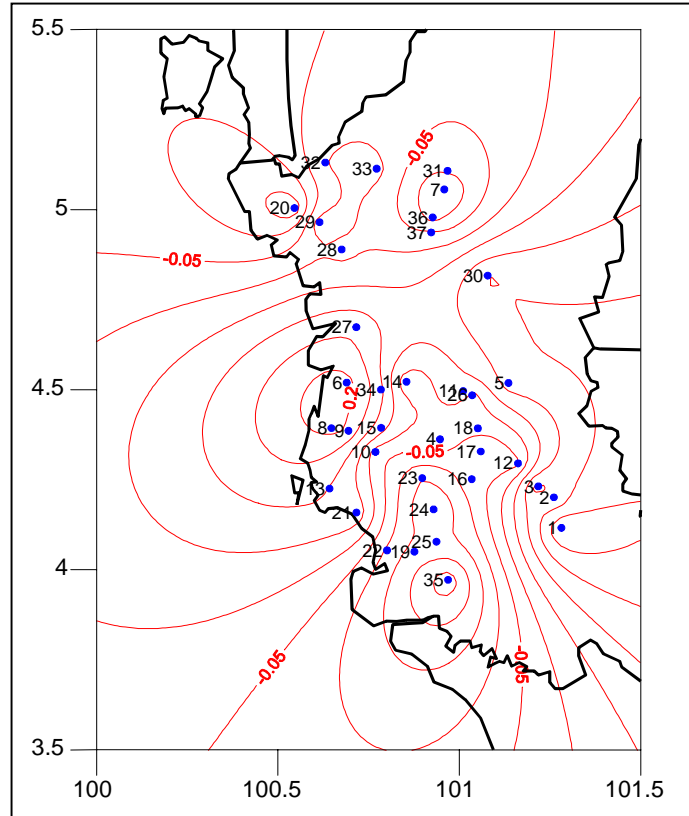
An in-house software was used to perform the least squares estimation and the result gives the following values:  $a_{00} = 0.1502\text{m}$ ,  $a_{01} = -13.3918$ ,  $a_{10} = 47.3674$  and  $a_{11} = -9.3516$ . This indicates that the local vertical datum is not physically parallel to the datum implied by the local geoid model by a shift of 15cm. It also shows that the shift and translation parameters are statistically significant for the area under study. Having applied the transformation, the statistics of the results is presented in Table 9.

From the Table , after the fit, using the four-parameter transformation model, the RMS of the residuals is at  $\pm 12.5\text{cm}$  and a standard deviation of  $\pm 13.4\text{cm}$ . The RMS difference before the fit, as indicated in Table 6, is  $\pm 21.8\text{cm}$  with a standard deviation of  $\pm 14.6\text{cm}$ . The contour plot of the residuals is illustrated in Figure 29. The significant improvement in the results after the application of bias and tilt indicate that the four-parameter model can be considered as adequate for the reduction of  $\delta N_i$  values.

However, results also indicate that at the current level of limitations of the geoid in particular, it cannot reach the millimeter level requirements of a first-order levelling operation. On a positive note though, the data set of GPS-derived geoidal heights can be a valuable resource for evaluating and testing the geoid model and/or vice-versa. This could also be another way of checking and identifying stations that may have Errors in their heights.

**Table 9** Results of 4-parameter transformation for 37 common points

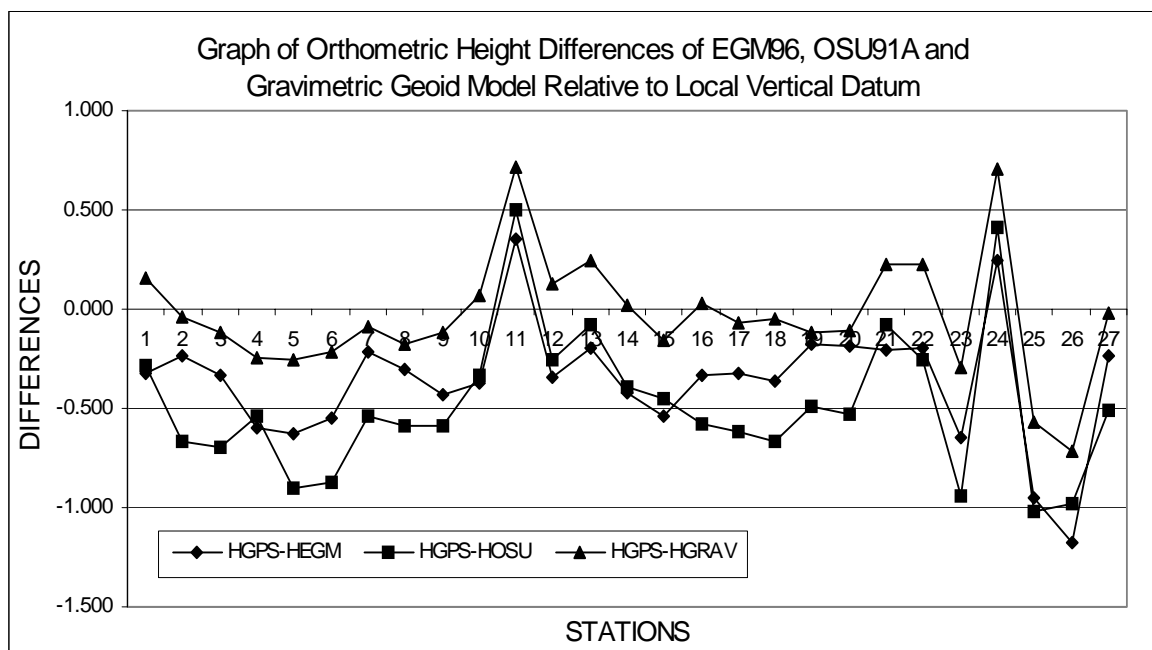
Station	Longitude $\lambda (^{\circ})$	Latitude $\phi (^{\circ})$	Residuals ( $v_i$ ) after transformation (m)
A0085	101.2814	4.1163	0.2503
A0089	101.2607	4.2010	0.0959
A0092	101.2179	4.2314	0.1822
A0123	100.9473	4.3623	-0.0051
A0152	101.1357	4.5184	0.1245
A0363	100.6897	4.5194	0.2186
A0424	100.9588	5.0557	-0.1553
A0500	100.6478	4.3934	0.2131
A0585	100.6946	4.3858	0.1914
A0600	100.7688	4.3268	-0.0288
A0635	101.0108	4.4959	0.0322
A0701	101.1621	4.2953	-0.0618
A0726	100.6425	4.2253	0.0967
A0832	100.8545	4.5221	-0.0374
A0840	100.7847	4.3941	0.0710
A0974	101.0345	4.2516	-0.0661
A0979	101.0592	4.3281	-0.1009
A0983	101.0511	4.3926	-0.0321
A1285	100.8766	4.0501	-0.1676
A1381	100.5459	5.0045	-0.1888
A1396	100.7165	4.1589	0.0898
A1555	100.8010	4.0537	-0.1085
A1597	100.8980	4.2543	-0.1496
A1601	100.9294	4.1672	-0.2053
A1606	100.9372	4.0780	-0.1462
A1622	101.0355	4.4842	-0.0227
A1802	100.7162	4.6735	0.1382
A1831	100.6757	4.8892	-0.0181
A1839	100.6150	4.9648	0.0343
S0091	101.0785	4.8167	0.1522
S0294	100.9676	5.1075	-0.0944
S0376	100.6309	5.1306	-0.0004
S0379	100.7725	5.1135	0.0162
S0411	100.7842	4.5000	0.1185
A0933	100.9695	3.9720	-0.2842
S0461	100.9268	4.9781	-0.1156
S0462	100.9227	4.9364	-0.0361
No. of points			37
Minimum ( $v_i$ )			-0.2842
Maximum ( $v_i$ )			0.2503
Mean ( $v_i$ )			0.0000
Std. Dev. ( $v_i$ )			$\pm 0.1337$
RMS ( $v_i$ )			$\pm 0.1253$



**Figure 28** Contour Plot of Residuals after bias and tilt fit with the four-parameter transformation model for 37 common points of GPS and levelling in Perak (shown as solid circles; contour interval is 0.05m)

### 3.3 Results of Investigation On Accuracy of GPS Heighting In The State of Johor

Figure 29 shows a plot of absolute orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum. Table 11 lists these differences. The results of Table 10 indicates that, the mean of these differences has values  $-36$  cm,  $-48$  cm and  $-3$  cm with uncertainties of 30 cm, 37 cm and 31 cm respectively. Figure 30 shows that there are large differences at stations 11, 24, 25 and 26. More investigations are required as to the causes of the large differences at these stations; the leveling and the GPS results should be checked carefully. These four stations are considered as outliers. These outliers may be due to Benchmark movement, ellipsoidal height error, and / or inaccuracies in the geoidal height model.



**Fig. 29.** Absolute orthometric height differences of EGM96, OSU91A and Gravimetric geoid models relative to local vertical datum

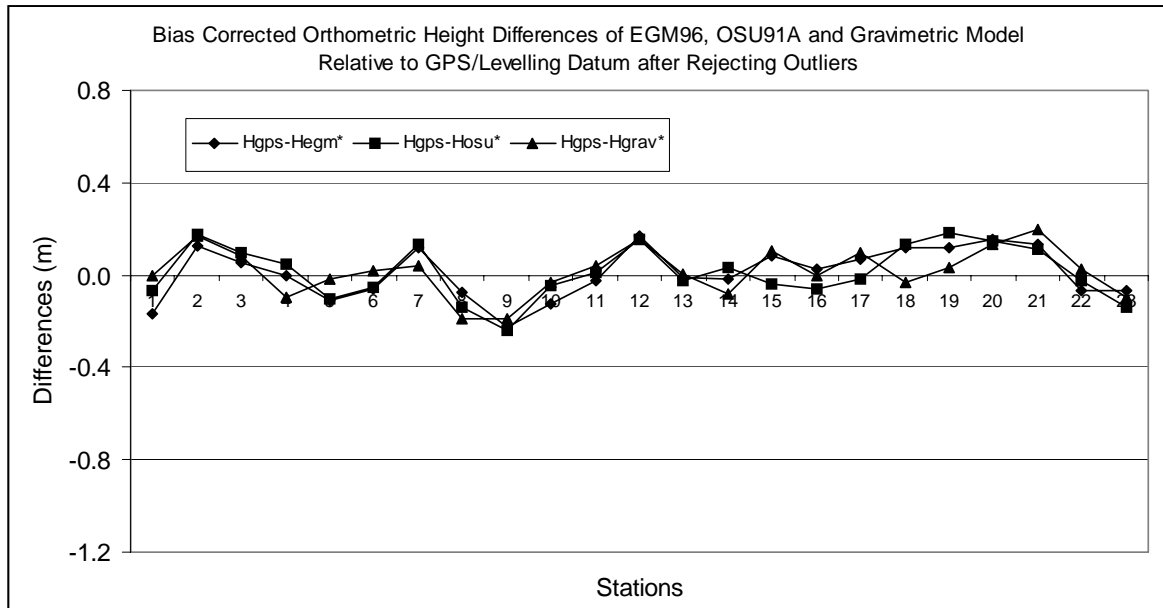
	$H_{GPS}-H_{EGM}$	$H_{GPS}-H_{OSU}$	$H_{GPS}-H_{GRAV}$
<i>MAXIMUM</i>	0.350	0.500	0.715
<i>MINIMUM</i>	-1.180	-1.020	-0.713
<i>MEAN</i>	<b>-0.359</b>	<b>-0.479</b>	<b>-0.031</b>
<i>RMS</i>	0.303	0.368	0.305
<i>STD</i>	0.297	0.361	0.299

**Table 10** Statistics of absolute orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum.

	$H_{GPS} - H_{EGM}^*$	$H_{GPS} - H_{OSU}^*$	$H_{GPS} - H_{GRAV}^*$
<i>MAXIMUM</i>	0.526	0.566	0.512
<i>MINIMUM</i>	-0.983	-0.903	-0.932
Mean	<b>0</b>	<b>0</b>	<b>0</b>
<i>RMS</i>	0.274	0.266	0.263
<i>STD. D</i>	0.269	0.261	0.258

**Table 11** Statistics of bias corrected absolute orthometric height differences of EGM96, OSU91A and gravimetric models relative to the local vertical datum

Figure 30 shows a plot of absolute orthometric height differences of the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum after rejecting stations 11, 24, 25 and 26 from the data set. These differences are listed in Table 12. The overall accuracies of these differences improved significantly, i.e., the uncertainties of the differences are 15 cm, 23 cm and 16 cm for the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum respectively.



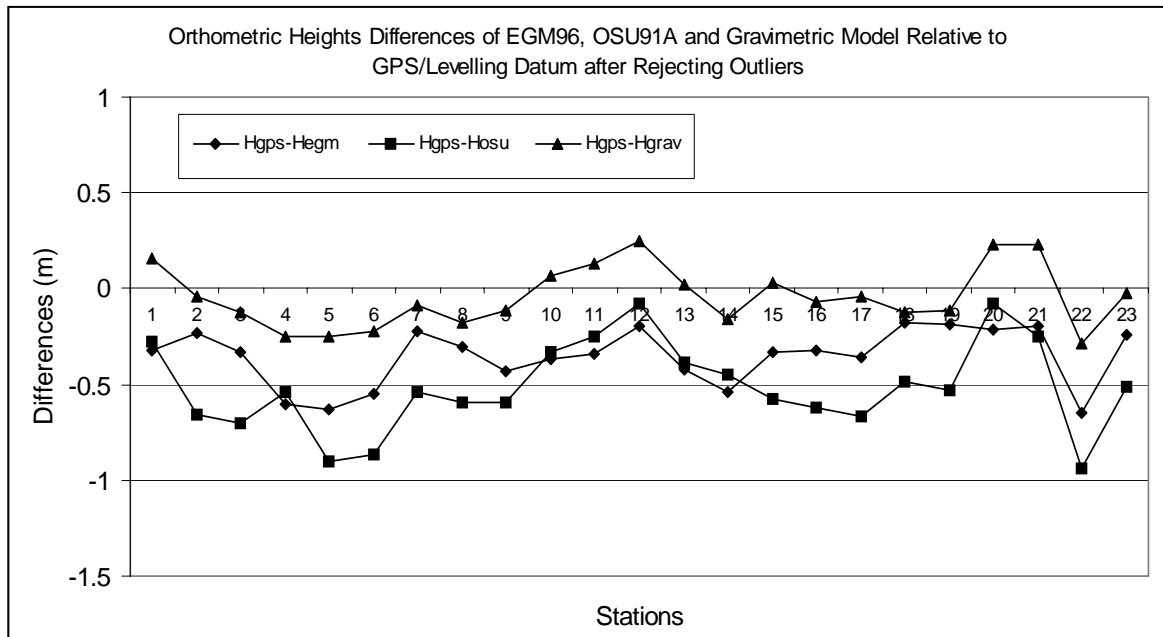
**Fig. 30.** Absolute bias corrected orthometric height differences of EGM96, OSU91A and Gravimetric geoid models relative to local vertical datum after rejecting outliers.

	$H_{GPS} - H_{EGM}$	$H_{GPS} - H_{OSU}$	$H_{GPS} - H_{GRAV}$
<i>MAXIMUM</i>	-0.180	-0.080	0.245
<i>MINIMUM</i>	-0.650	-0.940	-0.290
MEAN	<b>-0.355</b>	<b>-0.515</b>	<b>-0.042</b>
<i>RMS</i>	0.149	0.233	0.159
<i>STD. D</i>	0.145	0.228	0.156

**Table 12** Statistics of absolute orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum after rejection of outliers

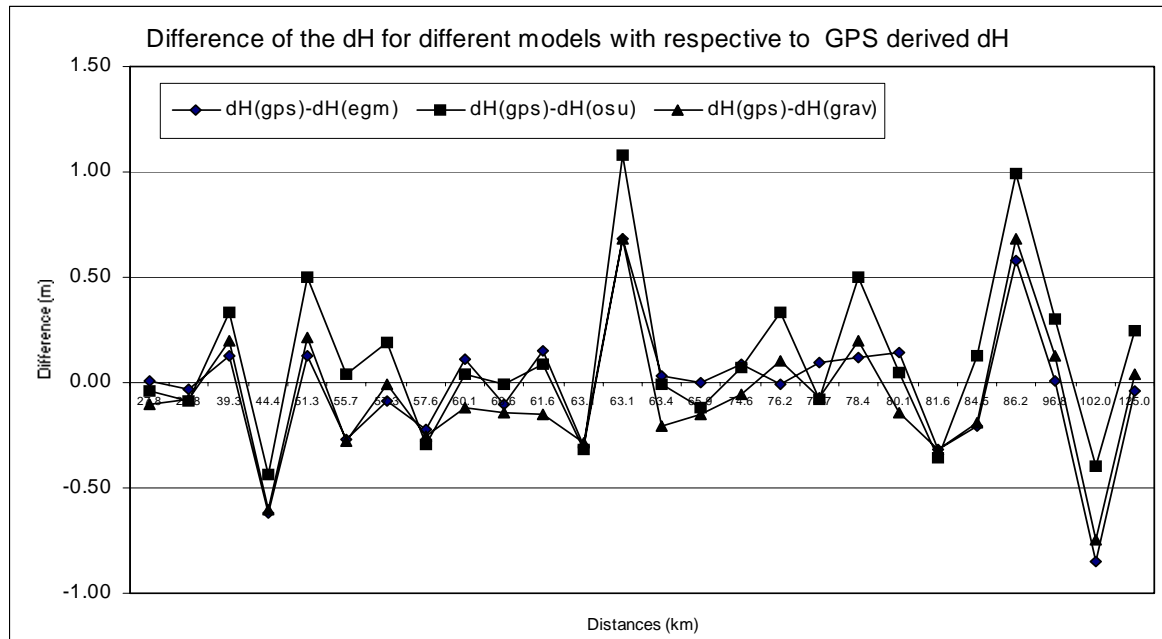
Figure 31 shows a plot of bias corrected absolute orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum. Table 12 lists these differences. Results of Table 11 suggests that, after a three-parameter trend of equation (6) was removed, the differences assumed zero means with uncertainties of 27 cm, 27 cm and 26 cm respectively.





**Fig. 31.** Absolute orthometric height differences of EGM96, OSU91A and Gravimetric geoid models relative to local vertical datum after rejecting outliers.

After rejection of outliers, the bias corrected absolute orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in Figure 32. These differences are listed in Table 13. The accuracy of these differences improved significantly, i.e., the uncertainties of the differences are 11 cm , 12 cm and 10 cm for the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum respectively.



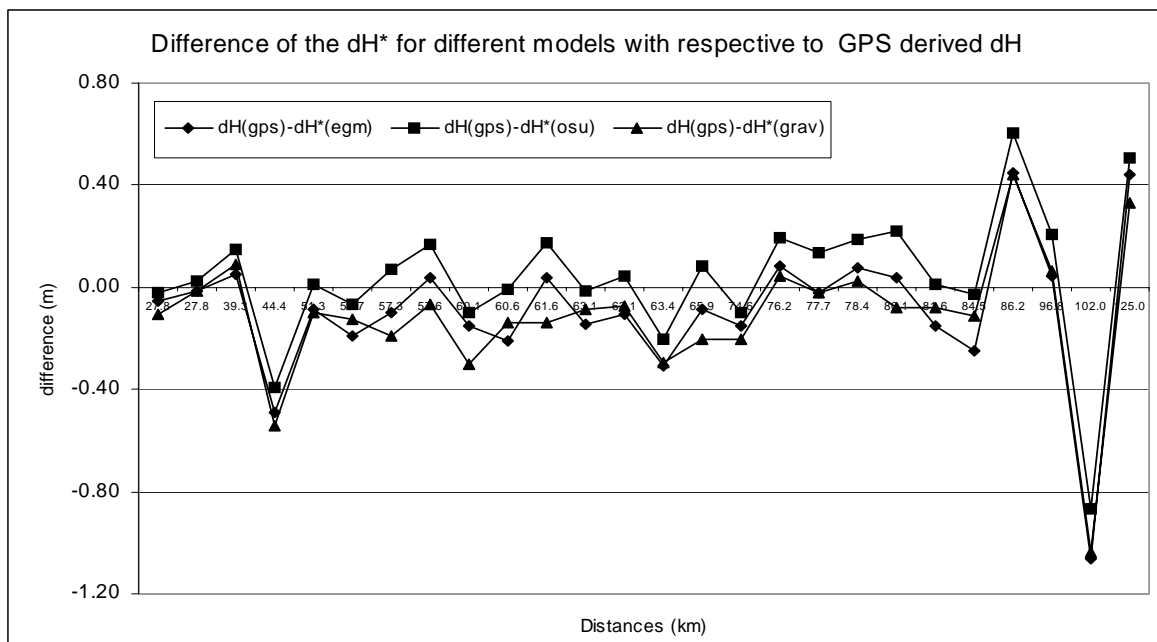
**Fig. 32.** Relative orthometric height differences derived from EGM96, OSU91A and Gravimetric model relative to GPS/Levelling datum (Station GP58 held fixed).

	$H_{GPS} - H_{EGM}^*$	$H_{GPS} - H_{OSU}^*$	$H_{GPS} - H_{GRAV}^*$
<i>MAXIMUM</i>	0.166	0.186	0.195
<i>MINIMUM</i>	-0.225	-0.240	-0.191
MEAN	0	0	0
<i>RMS</i>	0.110	0.115	0.104
<i>STD. D</i>	0.108	0.113	0.101

**Table 13** Statistics of bias corrected absolute orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum after rejection of Outliers.

Fixing station GP58, relative orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in Figure 33. These differences expressed in terms of part per million (ppm) are listed in Table 14. As seen in Table 14, the relative agreement derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum is about 14

to 0.2 ppm, 11 to 0.2 ppm and 14 to 0.2 ppm for baselines of 28 to 125 km respectively. The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 3.1, 3.9 and 3.5 for the above baseline lengths respectively.



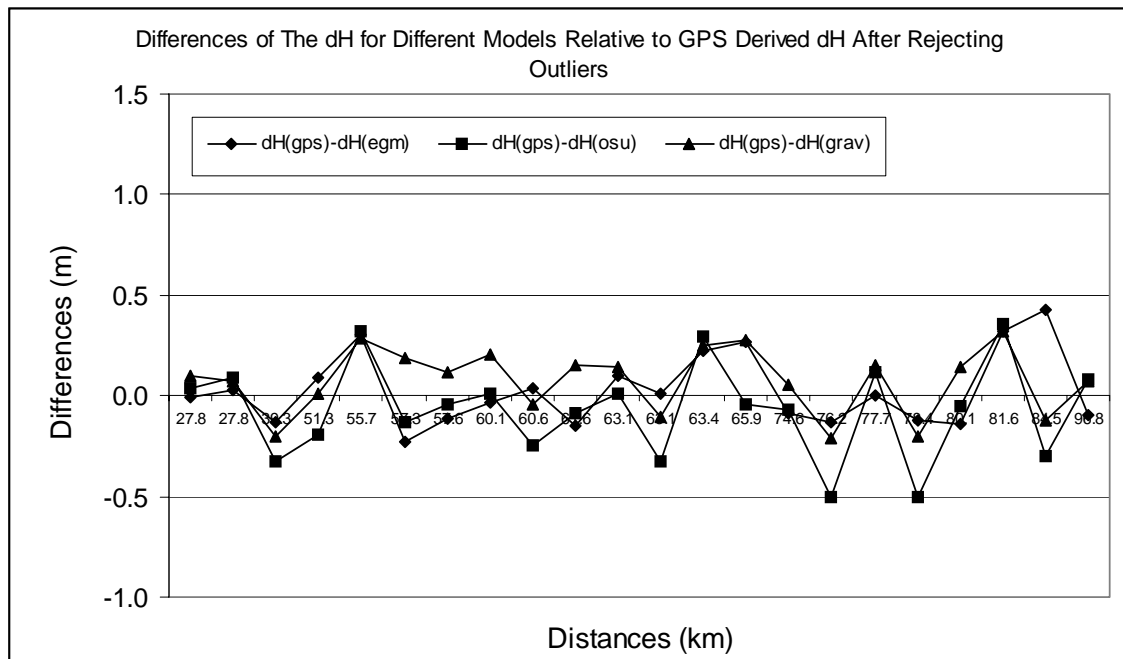
**Fig. 33.** Bias corrected relative orthometric height differences derived from EGM96, OSU91A and Gravimetric model relative to local vertical datum (Station GP58 held fixed).

ID	STN	$\Delta H_{GPS} - \Delta H_{EGM}$	ppm	$\Delta H_{GPS} - \Delta H_{OSU}$	ppm	$\Delta H_{GPS} - \Delta H_{GRAV}$	ppm	Distance (km)
1	GP58	<b>fixed</b>		<b>fixed</b>		<b>fixed</b>		
2	GP15	-0.098	1.012	0.082	0.847	0.073	0.758	96.8
3	GP16	0.000	0.000	0.120	1.544	0.151	1.944	77.7
4	GP43	0.270	4.095	-0.040	0.607	0.276	4.184	65.9
5	GP44	0.300	5.383	0.320	5.742	0.283	5.076	55.7
6	GP47	0.220	3.488	0.290	4.598	0.250	3.968	63.1
7	GP48	-0.110	1.909	-0.040	0.694	0.116	2.007	57.6
8	GP49	-0.030	0.499	0.010	0.166	0.204	3.387	60.1
9	GP50	0.100	1.578	0.010	0.158	0.144	2.276	63.4
10	GP51	0.040	0.660	-0.250	4.128	-0.040	0.652	60.6
11	GP52	-0.680	5.440	-1.080	8.640	-0.685	5.483	125.0
12	GP53	0.010	0.158	-0.330	5.229	-0.101	1.600	63.1
13	GP54	-0.130	1.705	-0.500	6.559	-0.215	2.822	76.2
14	GP55	0.090	1.756	-0.190	3.707	0.007	0.146	51.3
15	GP56	-0.230	4.014	-0.130	2.269	0.189	3.293	57.3
16	DOP2	0.430	5.087	-0.300	3.549	-0.126	1.488	84.5
17	GP59	-0.010	0.360	0.040	1.441	0.103	3.697	27.8
18	GP61	0.030	1.081	0.090	3.243	0.076	2.724	27.8
19	GP84	-0.150	2.436	-0.090	1.462	0.152	2.475	61.6
20	GP85	-0.140	1.749	-0.050	0.625	0.142	1.779	80.1
21	GP90	-0.120	1.531	-0.500	6.380	-0.200	2.551	78.4
22	GP91	-0.130	3.310	-0.330	8.402	-0.200	5.095	39.3
23	TG05	0.320	3.919	0.360	4.409	0.320	3.918	81.6
24	TG07	-0.580	6.728	-0.990	11.484	-0.680	7.882	86.2
25	TG09	0.620	13.977	0.440	9.919	0.600	13.531	44.4
26	TG10	0.850	8.329	0.400	3.920	0.743	7.283	102.0
27	TG19	-0.090	1.207	-0.070	0.939	0.053	0.715	74.6
	<b>Mean</b>	-0.030	<b>3.131</b>	0.105	<b>3.872</b>	-0.063	<b>3.490</b>	
	<b>Sd</b>	0.321	<b>3.091</b>	0.374	<b>3.189</b>	0.311	<b>2.817</b>	

**Table 14** Relative orthometric height differences of EGM96, OSU91A and the Gravimetric model relative to the local vertical datum expressed in part per million (Station GP58 held fixed)

After rejection of outliers and fixing station GP58, relative orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in Figure 34. These differences expressed in terms of ppm are listed in Table 15. As seen in Table 15, the relative agreement of the EGM96, OSU91A

and the gravimetric geoid model relative to the local vertical datum is about 5 to 0.2 ppm, 8 to 0.2 ppm and 5 to 0.2 ppm for baselines of 28 to 125 km respectively. The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 2.1, 3.0 and 2.6 for the above baseline lengths respectively.

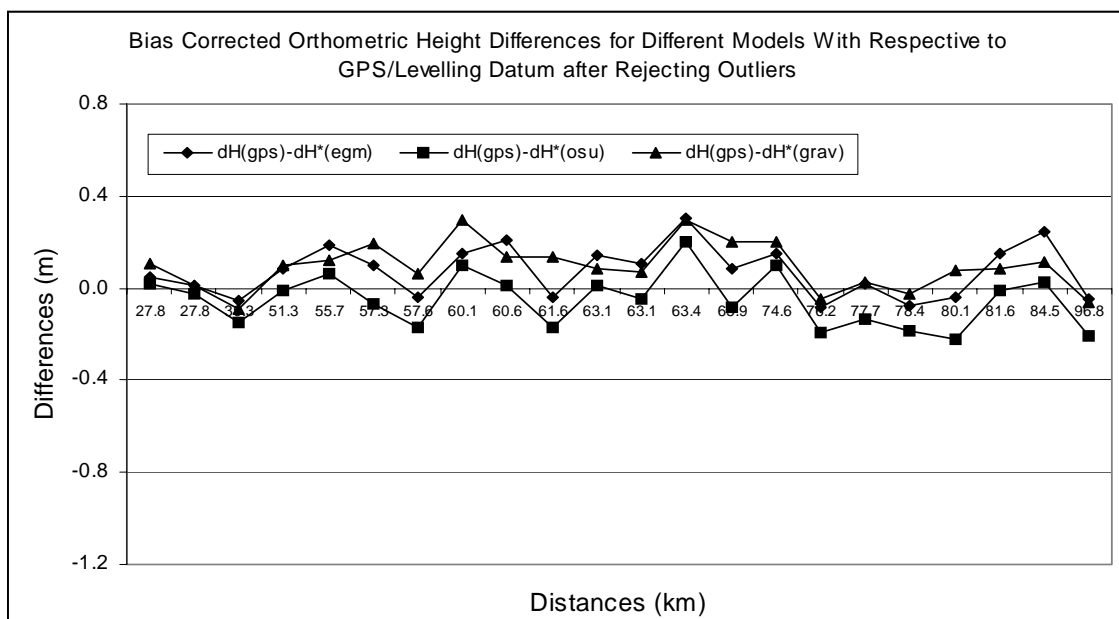


**Fig. 34.** Bias corrected relative orthometric height differences of EGM96, Osu91A and Gravimetric model after rejecting outliers.

ID	STN	$\Delta H_{GPS} - \Delta H_{EGM}$	ppm	$\Delta H_{GPS} - \Delta H_{OSU}$	ppm	$\Delta H_{GPS} - \Delta H_{GRAV}$	ppm
1	GP58	<b>fixed</b>		<b>fixed</b>		<b>fixed</b>	
2	GP15	-0.098	1.012	0.082	0.847	0.073	0.758
3	GP16	0.000	0.000	0.120	1.544	0.151	1.944
4	GP43	0.270	4.095	-0.040	0.607	0.276	4.184
5	GP44	0.300	5.383	0.320	5.742	0.283	5.076
6	GP47	0.220	3.488	0.290	4.598	0.250	3.968
7	GP48	-0.110	1.909	-0.040	0.694	0.116	2.007
8	GP49	-0.030	0.499	0.010	0.166	0.204	3.387
9	GP50	0.100	1.578	0.010	0.158	0.144	2.276
10	GP51	0.040	0.660	-0.250	4.128	-0.040	0.652
12	GP53	0.010	0.158	-0.330	5.229	-0.101	1.600
13	GP54	-0.130	1.705	-0.500	6.559	-0.215	2.822
14	GP55	0.090	1.756	-0.190	3.707	0.007	0.146
15	GP56	-0.230	4.014	-0.130	2.269	0.189	3.293
16	DOP2	0.430	5.087	-0.300	3.549	-0.126	1.488
17	GP59	-0.010	0.360	0.040	1.441	0.103	3.697
18	GP61	0.030	1.081	0.090	3.243	0.076	2.724
19	GP84	-0.150	2.436	-0.090	1.462	0.152	2.475
20	GP85	-0.140	1.749	-0.050	0.625	0.142	1.779
21	GP90	-0.120	1.531	-0.500	6.380	-0.200	2.551
22	GP91	-0.130	3.310	-0.330	8.402	-0.200	5.095
23	TG05	0.320	3.919	0.360	4.409	0.320	3.918
27	TG19	-0.090	1.207	-0.070	0.939	0.053	0.715
		0.026	<b>2.134</b>	-0.068	<b>3.032</b>	0.075	<b>2.571</b>
		0.180	<b>1.604</b>	0.238	<b>2.406</b>	0.162	<b>1.404</b>

**Table 15** Relative orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum expressed in parts per million after rejecting outliers (Station GP58 held fixed)

Fixing station GP58, bias corrected relative orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in figure 35. These differences expressed in terms of part per million (ppm) are listed in Table 16. As seen in Table 17, the relative agreement of the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum is about 11 to 0.3 ppm, 9 to 0.2 ppm and 12 to 0.3 ppm for baselines of 28 to 125 km respectively.



**Fig. 35.** Relative orthometric height of EGM96, OSU91A and Gravimetric model relative differences after rejecting outliers.

ID	STN	$\Delta H_{GPS} - \Delta H_{EGM}^*$	ppm	$\Delta H_{GPS} - \Delta H_{OSU}^*$	ppm	$\Delta H_{GPS} - \Delta H_{GRAV}^*$	ppm	Distance (km)
1	GP58	<b>fixed</b>		<b>fixed</b>		<b>fixed</b>		
2	GP15	-0.044	0.455	-0.210	2.169	-0.065	0.667	96.8
3	GP16	0.022	0.283	-0.136	1.750	0.024	0.310	77.7
4	GP43	0.085	1.289	-0.086	1.304	0.205	3.107	65.9
5	GP44	0.188	3.373	0.065	1.166	0.122	2.187	55.7
6	GP47	0.142	2.251	0.013	0.206	0.086	1.368	63.1
7	GP48	-0.041	0.711	-0.170	2.950	0.066	1.140	57.6
8	GP49	0.154	2.563	0.102	1.698	0.298	4.968	60.1
9	GP50	0.305	4.814	0.203	3.204	0.297	4.691	63.4
10	GP51	0.207	3.418	0.011	0.182	0.139	2.287	60.6
11	GP52	-0.443	3.544	-0.510	4.080	-0.328	2.627	125
12	GP53	0.107	1.695	-0.047	0.745	0.071	1.125	63.1
13	GP54	-0.086	1.128	-0.195	2.558	-0.045	0.592	76.2
14	GP55	0.087	1.697	-0.011	0.215	0.101	1.961	51.3
15	GP56	0.097	1.693	-0.069	1.204	0.193	3.363	57.3
16	DOP2	0.248	2.934	0.029	0.343	0.111	1.315	84.5
17	GP59	0.051	1.838	0.021	0.757	0.109	3.914	27.8
18	GP61	0.014	0.505	-0.022	0.793	0.014	0.490	27.8
19	GP84	-0.039	0.633	-0.173	2.810	0.137	2.232	61.6
20	GP85	-0.039	0.487	-0.223	2.785	0.077	0.967	80.1
21	GP90	-0.077	0.983	-0.188	2.399	-0.026	0.330	78.4
22	GP91	-0.054	1.375	-0.151	3.845	-0.087	2.218	39.3
23	TG05	0.149	1.825	-0.013	0.159	0.082	1.003	81.6
24	TG07	-0.446	5.173	-0.603	6.995	-0.444	5.144	86.2
25	TG09	0.491	11.069	0.394	8.882	0.543	12.246	44.4
26	TG10	1.063	10.417	0.866	8.486	1.040	10.193	102
27	TG19	0.149	1.998	0.101	1.354	0.203	2.726	74.6
		-0.088	<b>2.621</b>	0.039	<b>2.425</b>	-0.112	<b>2.814</b>	
	<b>Sd</b>	0.279	<b>2.720</b>	0.271	<b>2.403</b>	0.267	<b>2.862</b>	

**Table 16.** Bias corrected relative orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum expressed in parts per million (Station GP58 held fixed)



ID	STN	$h$	$\Delta h$	$H^*$	$\Delta H^*$	$N^*$	$\Delta N^*$
1	GP58	77.620	<b>fixed</b>	71.993	<b>fixed</b>	5.628	<b>fixed</b>
2	GP15	4.002	73.618	1.787	70.206	2.215	3.413
3	GP16	36.230	41.390	33.386	38.607	2.844	2.784
4	GP43	11.180	66.440	5.087	66.906	6.093	-0.465
5	GP44	55.650	21.970	51.884	20.109	3.766	1.862
6	GP47	51.180	26.440	47.838	24.155	3.342	2.286
7	GP48	164.750	-87.130	160.948	-88.955	3.802	1.826
8	GP49	7.360	70.260	2.141	69.852	5.220	0.409
9	GP50	10.120	67.500	3.999	67.994	6.121	-0.493
10	GP51	42.710	34.910	35.551	36.442	7.160	-1.532
11	GP52	67.810	9.810	58.284	13.709	9.526	-3.898
12	GP53	31.820	45.800	23.963	48.030	7.857	-2.229
13	GP54	12.880	64.740	4.507	67.486	8.373	-2.745
14	GP55	36.740	40.880	29.323	42.670	7.418	-1.790
15	GP56	11.570	66.050	4.765	67.228	6.805	-1.177
16	DOP2	90.910	-13.290	83.803	-11.810	7.107	-1.479
17	GP59	45.180	32.440	40.151	31.842	5.029	0.599
18	GP61	41.830	35.790	37.226	34.767	4.604	1.024
19	GP84	7.130	70.490	3.179	68.814	3.951	1.677
20	GP85	4.930	72.690	1.849	70.144	3.081	2.547
21	GP90	10.780	66.840	2.336	69.657	8.444	-2.816
22	GP91	26.310	51.310	19.355	52.638	6.955	-1.327
23	TG05	193.730	-116.110	190.774	-118.781	2.956	2.672
24	TG07	194.270	-116.650	185.649	-113.656	8.622	-2.994
25	TG09	223.000	-145.380	217.135	-145.142	5.865	-0.237
26	TG10	193.250	-115.630	184.432	-112.439	8.818	-3.190
27	TG19	136.070	-58.450	130.495	-58.502	5.575	0.053

**Table 17** Bias corrected Relative differences derived from gravimetric model and GPS/leveling data (Station GP58 held fixed)

The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 2.6, 2.4 and 2.8 for the above baseline lengths respectively.

After rejection of outliers and fixing station GP58, bias corrected relative orthometric height differences derived from the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum are depicted in figure 33. These differences expressed

in terms of ppm are listed in Table 18. As seen in Table 33, the relative agreement of the EGM96, OSU91A and the gravimetric geoid model relative to the local vertical datum is about 5 to 0.3 ppm, 4 to 0.2 ppm and 5 to 0.3 ppm for baselines of 28 to 125 km respectively. The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 1.7, 1.6 and 2.0 for the above baseline lengths respectively

ID	STN	$\Delta H_{GPS} - \Delta H_{EGM}^*$	ppm	$\Delta H_{GPS} - \Delta H_{OSU}^*$	ppm	$\Delta H_{GPS} - \Delta H_{GRAV}^*$	ppm
1	GP58	<b>fixed</b>		<b>Fixed</b>		<b>fixed</b>	
2	GP15	-0.044	0.455	-0.210	2.169	-0.065	0.667
3	GP16	0.022	0.283	-0.136	1.750	0.024	0.310
4	GP43	0.085	1.289	-0.086	1.304	0.205	3.107
5	GP44	0.188	3.373	0.065	1.166	0.122	2.187
6	GP47	0.142	2.251	0.013	0.206	0.086	1.368
7	GP48	-0.041	0.711	-0.170	2.950	0.066	1.140
8	GP49	0.154	2.563	0.102	1.698	0.299	4.968
9	GP50	0.305	4.814	0.203	3.204	0.297	4.691
10	GP51	0.207	3.418	0.011	0.182	0.139	2.287
12	GP53	0.107	1.695	-0.047	0.745	0.071	1.125
13	GP54	-0.086	1.128	-0.195	2.558	-0.045	0.592
14	GP55	0.087	1.697	-0.011	0.215	0.101	1.961
15	GP56	0.097	1.693	-0.069	1.204	0.193	3.363
16	DOP2	0.248	2.934	0.029	0.343	0.111	1.315
17	GP59	0.051	1.838	0.021	0.757	0.109	3.914
18	GP61	0.014	0.505	-0.022	0.793	0.014	0.490
19	GP84	-0.039	0.633	-0.173	2.810	0.137	2.232
20	GP85	-0.039	0.487	-0.223	2.785	0.077	0.967
21	GP90	-0.077	0.983	-0.188	2.399	-0.026	0.330
22	GP91	-0.054	1.375	-0.151	3.845	-0.087	2.218
23	TG05	0.149	1.825	-0.013	0.159	0.082	1.003
27	TG19	0.149	1.998	0.101	1.354	0.203	2.726
		0.074	<b>1.725</b>	-0.052	<b>1.573</b>	0.096	<b>1.953</b>
		0.112	<b>1.151</b>	0.118	<b>1.119</b>	0.104	<b>1.373</b>

**Table 18** Bias corrected relative orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum expressed in parts per million after rejecting outliers (Station GP58 held fixed).

Results of Table 19 indicates that local vertical datum is not physically parallel with the EGM96, OSU91A and the gravimetric geoid. The shift parameters between the vertical datum implied by the GPS/leveling datum, and the EGM96, OSU91A and the gravimetric datums are about -41 cm, -54 cm and -8 cm respectively. Also the maximum tilts of the planes fitting the residual geoids above these datums relative to GPS/leveling datum are of the order of 36, 51 and 33 centimeters per degree. It is therefore necessary to take into consideration the effect of inconsistent datum bias particularly for long baseline height transfer.

Residual geoid Cm	$c_1$ cm/degree	$c_2$ cm/degree	$c_0$ cm	Tilt cm/degree	Azimuth degrees
$N_{GPS} - N_{EGM96}$	-35.8	-3.7	-41.0	36.0	5.90
$N_{GPS} - N_{OSU91A}$	-35.8	35.9	-54.0	50.7	134.93
$N_{GPS} - N_{GRAV}$	-27.7	17.8	-7.8	32.9	147.28

**Table 19** Datum shift parameters of EGM96, OSU91A and gravimetric geoids relative to the local vertical datum (GPS/leveling datum)

When analyzing these absolute errors and the relative errors that follow, it should be kept in mind that these are not purely leveling errors, but contain gravimetric geoid determination errors and GPS position errors as well. Consequently, they could also be used to identify stations that have errors in their ellipsoidal or geoidal heights.

A comparison of different geoid models, as seen from all the tables and figures, indicates that the OSU91A model had the best agreement with the GPS/leveling data when comparing orthometric height differences in an absolute mode. While the EGM96 geoid had the best agreement with the GPS/leveling data when comparing orthometric height differences in a relative mode. The regional gravimetric geoid had the least agreement

with GPS/leveling data. This may be due to poor gravity coverage in the test area. Also in our regional geoid solution, the terrain effect on the anomalies as well as their by product indirect effect on the computed geoid was not considered.

Using bias corrected relative orthometric height differences derived from the EGM96, OSU91A and the regional gravimetric geoid models combined with GPS/leveling data, we can achieve relative accuracy of the order of 1.7, 1.6 and 2 ppm for baseline lengths of 28-125 km respectively. For a baseline length of 36 km, the mean accuracy of these bias corrected relative orthometric height differences derived from the EGM96, OSU91A and the regional gravimetric geoid models are 61 , 58 and 72 mm respectively. The precision of third order leveling based on the Malaysian height datum for the same baseline length is:  $12\sqrt{K}=12\sqrt{36}=72$  mm. This means, the level of accuracy achieved by the bias corrected relative orthometric height differences of the above three models for baseline lengths up to 36 km, is sufficient to replace the conventional tedious, time consuming ordinary leveling technique for rapid height transfer for land surveying and engineering applications. The other tables and figures relate to the result of this research area can also be read in the appendix.

## 4.0 CONCLUSIONS

### 4.1 Conclusion For Peninsular Malaysia Research Area

Significant progress has been made over the last decade since the initiation of the geoid project in Peninsular Malaysia. This progress can be seen in terms of the gravity data collection (from 2000 points in 1989 to over 10000 points in 2000), the computation algorithm (Stokes integral, LSC, FFT) and the evaluation of results (use of GPS data, satellite altimetry).

A preliminary gravimetric geoid model, MYGeoid02, of the Peninsular Malaysia and its surrounding marine area, covering an area  $0^{\circ} 00' N \leq \phi \leq 8^{\circ} 00' N$  and  $99^{\circ} 00' E \leq \lambda \leq 105^{\circ} 00' E$  was computed and presented. MYGeoid02 was computed using the following data: (1) a global geopotential model, EGM96 spherical harmonic coefficients set complete to degree and order 360, (2) a set of 10400 point free-air gravity anomalies belonging to the DSMM Gravity Data Bank and referred to the GRS80, (3) satellite altimetry-derived geoid heights on a 1.5' by 1.5' grid, extracted from a database of the Danish National Survey and (4) a digital terrain data set from the 30'' point topography database, GTOPO. The method applied to obtain the geoid was the Stokes integral in convolution form using gridded gravity anomalies as the input data. The geoid was computed by means of a 1-D FFT convolution about EGM96 spherical harmonic model of the geopotential and based on the "remove-compute-restore" technique. This technique was used, as it is conceptually simple and easy to implement. A 100% zero padding was appended around the signal matrix to avoid circular convolution effects and terrain correction was also applied to the data.

Both geoids over Peninsular Malaysia and the adjacent marine areas, EGM96 and MYGeoid02, are rising from north-west to south-east. They coincide with the reference ellipsoid GRS80 roughly in the central region, from  $102^{\circ}E$  meridian to  $103.5^{\circ}E$  meridian

and from 2°N to 5°N. Estimates of the geoidal heights range from 10m in the south to -15m in the north. Results also indicate that the maximum and minimum values of the MYGeoid02 are 9.45m and -14.66m respectively. Whilst for the EGM96 geoid, the figures are 9.53m and -14.47m. MYGeoid02 and EGM96 have a mean undulation value of 9.45m and 9.53m respectively.

The computed geoid, MYGeoid02 and also EGM96 geoid for Peninsular Malaysia were compared with GPS/levelling derived geoid. MYGeoid02 seems to show a slight improvement in precision. The results are presented in the form of contour maps. Similar comparisons were made with EGM96 global geoid. Comparison with geoid undulations obtained by GPS/MSL shows the accuracies of MYGeoid02 and EGM96 are 0.39m and 0.40m respectively. Thus it can be concluded that in general MYGeoid02 is slightly the better model for the transformation of GPS-derived ellipsoidal heights to orthometric heights as compared to EGM96. However, it is anticipated that the accuracy will improve further with the continuing efforts by the DSMM in collecting more gravity points in its monthly routine. Also in the pipeline is the use of airborne gravimetry project, which will begin in 2002.

The advent of GPS has increased the need for an accurate knowledge of geoidal heights. However, the application of the GPS technique for practical levelling is an attractive option provided that the geoid is known precisely enough. With the full constellation of 24 satellites operational that enables excellent geometry and the removal of selective availability, the need to use GPS for levelling can be realised.

## **4.2 Conclusion of The Detailed Study of Datum Bias In The State of Perak**

Operational methodologies and accuracy of GPS levelling as demonstrated by the case study in Perak reveal the potential of GPS in height surveys as encouraging. Differences between the computed local geoid and the observed levelled heights were determined for the test area. These differences are found to be quite inhomogeneous, even for stations that are close to each other. However using the derived local geoid, the differences are decreased and tend to be more homogeneous. It can also be noticed that the differences increase in the North-south direction. This is due to the sparse gravity data coverage in the south and may also due to the gravity measurement reduction errors. Comparison between the computed geoidal height differences and the GPS-derived values reveals accuracies of 8.0 to 9.9 ppm can be obtained for average baseline length of 13 km. GPS heighting is generally more cost-effective than the standard levelling techniques. However the accuracy, as the case study has demonstrated, appears to be at best of the order of 8ppm. As such, GPS heighting can provide 2<sup>nd</sup> order levelling standard ( $12\text{mm}/\sqrt{K}$ ) only for distances of less than 2.5km. For 1<sup>st</sup> order levelling where the required accuracies are at sub-cm level, the present method is not a feasible alternative to traditional levelling. However, it can be suggested that, with a ‘hypothetical’ cm-accuracy geoid, the use of GPS in height surveys is comparable to 1<sup>st</sup> order levelling over lines not more than 2km and to 2<sup>nd</sup> order levelling for distances up to 30km.

Thus, depending on the intended application, GPS technology when used together with a geoid height model can function as a levelling system. Although the traditional spirit levelling has been a technique of choice in the determination of orthometric heights, its main deficiencies are its labour-intensive and time-consuming operation. On the other hand, the local accuracy of a geoid model used in GPS heighting is dependant on the accuracy on the gravity measurements. The coverage or the density of gravity data used in gravimetric geoid determination is paramount. Of equal importance too is the DEM that is used to produce the mean gravity grid.

The present method of GPS heighting, as the research in Perak area has indicated, is not a feasible alternative to the first-order spirit levelling. The presence of systematic errors in GPS heighting produces tilts and biases to the geoid model, whilst gross errors give rise to punctual deformation to the geoid model. The minimisation of the residuals can be carried out quite adequately using a four-parameter transformation model in order to absorb the vertical datum inconsistencies. Results from the Perak test area indicate that a bias of 15cm could exist in the GPS geoid model and that once this bias is removed, the RMS fit of the residuals is 12.5cm. Considering the RMS difference between the GPS and gravimetric geoids before the fit was 21.8cm, the overall improvement can be regarded as significant.

The data set of GPS-derived geoidal heights can be a valuable resource for evaluating and testing the geoid model in any area of interest and vice-versa. This in turn could help in checking and identifying stations that may have errors in their heights. Apart from the need to have a more precise geoid, the accuracy of the ellipsoidal heights from GPS could be improved by using precise ephemerides, especially for longer baselines. It is only then that the accuracy of GPS height surveys be increased once these error contributions are lowered. This would finally pave the way to successfully combine the measurements of horizontal position and height into one complete technique, which presently in Peninsular Malaysia and in most parts of the world still remain as two separate entities



### **4.3 Conclusion of The Investigation On Accuracy of GPS Heighting In The State of Johor**

An important aspect of any geodetic positioning technique is to ensure that all outliers have been removed from the data. GPS heighting is a viable alternative for checking and validating ellipsoidal heights and orthometric heights of known Benchmarks. The accuracy of the GPS-derived orthometric height differences depends on the accuracy of the GPS-derived ellipsoidal height differences and the accuracy of the geoid model considered in the investigation. Efforts to improve the accuracies of geoid height differences will depend on overall national accuracy needs for GPS-derived orthometric height differences and on the cost of differential leveling versus GPS and gravity survey methods.

GPS measurements combined with accurate geoid models can be used for successful rapid height transfer. The investigation of quality of the GPS-derived bias corrected relative orthometric heights is vital for the community of surveyors and oceanographers. Regional geoid models accurate to a sub—decimeter level would be widely used, e.g. by surveyors to rigorously transform the results of GPS positioning to a local datum.

The relative agreement of the EGM96, OSU91A and the gravimetric geoid models relative to the local vertical datum is about 5 to 0.3, 4 to 0.2 and 5 to 0.3 ppm for baselines length of 28 to 125 km respectively. The average agreement in terms of ppm of the above three models relative to GPS/leveling datum is of the order of 1.7, 1.6 and 2 ppm respectively.

Within third order leveling specifications based on the Malaysian height datum, the level of accuracy achieved by the bias corrected relative orthometric height differences of the EGM96, OSU91A and the gravimetric geoid models combined with GPS/leveling data for baseline lengths up to 36 km, is sufficient to replace the conventional tedious, time consuming ordinary leveling technique for rapid height transfer for land surveying and engineering applications.

#### **4.4 Overall Conclusions**

- Local Vertical Datum is not consistent w.r.t Geoid Models. It means that GPS derived Orthometric Height does not fit Local MSL Height.
- Inconsistencies due to: ellipsoidal or GPS height determination, Geoid Model, and Errors in BM.
- More studies to be done : Using newly derived Gravimetric Geoid and Monitoring the status of BM.

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## APPENDIX

ST. NO	NAME	$\phi$	$\lambda$	H	H
1	DOP2	1.380	103.610	90.910	83.800
2	GP15	2.060	102.560	4.002	1.960
3	GP16	2.130	102.730	36.230	33.470
4	GP43	2.600	103.780	11.180	4.990
5	GP44	2.470	103.070	55.650	51.870
6	GP47	2.390	102.930	51.180	47.860
7	GP48	1.980	102.930	164.750	160.990
8	GP49	1.630	103.200	7.360	1.950
9	GP50	1.550	103.400	10.120	3.810
10	GP51	1.630	103.670	42.710	35.520
11	GP52	1.370	104.270	67.810	58.720
12	GP53	1.800	103.900	31.820	24.000
13	GP54	1.930	104.090	12.880	4.660
14	GP55	2.080	103.890	36.740	29.330
15	GP56	2.390	103.870	11.570	4.680
16	GP58	2.120	103.430	77.620	72.100
17	GP59	1.970	103.230	45.180	40.150
18	GP61	2.190	103.190	41.830	37.320
19	GP84	1.860	102.940	7.130	3.150
20	GP85	1.910	102.740	4.930	1.880
21	GP90	1.930	104.110	10.780	2.470
22	GP91	1.880	103.690	26.310	19.550
23	TG05	2.650	102.920	193.730	190.800
24	TG07	1.680	104.070	194.270	186.200
25	TG09	2.460	103.640	223.000	216.700
26	TG10	1.460	104.070	193.250	183.500
27	TG19	1.470	103.260	136.070	130.400

**Table 20** Johor GPS data associated with leveling.

ST. NO	NAME	$\phi$	$\lambda$	$N_{GPS}$	$N_{EGM}$	$N_{OSU}$	$N_{GRAV}$
1	DOP2	1.380	103.610	7.1100	6.7900	6.8300	7.2658
2	GP15	2.060	102.560	2.0420	1.8100	1.3800	1.9986
3	GP16	2.130	102.730	2.7600	2.4300	2.0600	2.6389
4	GP43	2.600	103.780	6.1900	5.5900	5.6500	5.9441
5	GP44	2.470	103.070	3.7800	3.1500	2.8800	3.5271
6	GP47	2.390	102.930	3.3200	2.7700	2.4500	3.0997
7	GP48	1.980	102.930	3.7600	3.5400	3.2200	3.6743
8	GP49	1.630	103.200	5.4100	5.1100	4.8200	5.2365
9	GP50	1.550	103.400	6.3100	5.8800	5.7200	6.1958
10	GP51	1.630	103.670	7.1900	6.8200	6.8600	7.2595
11	GP52	1.370	104.270	9.0900	9.4400	9.5900	9.8054
12	GP53	1.800	103.900	7.8200	7.4800	7.5700	7.9510
13	GP54	1.930	104.090	8.2200	8.0200	8.1400	8.4651
14	GP55	2.080	103.890	7.4100	6.9900	7.0200	7.4325
15	GP56	2.390	103.870	6.8900	6.3500	6.4400	6.7313
16	GP58	2.120	103.430	5.5200	5.1900	4.9400	5.5495
17	GP59	1.970	103.230	5.0300	4.7100	4.4100	4.9574
18	GP61	2.190	103.190	4.5100	4.1500	3.8400	4.4644
19	GP84	1.860	102.940	3.9800	3.8000	3.4900	3.8576
20	GP85	1.910	102.740	3.0500	2.8600	2.5200	2.9376
21	GP90	1.930	104.110	8.3100	8.1000	8.2300	8.5399
22	GP91	1.880	103.690	6.7600	6.5600	6.5100	6.9901
23	TG05	2.650	102.920	2.9300	2.2800	1.9900	2.6401
24	TG07	1.680	104.070	8.0700	8.3200	8.4800	8.7795
25	TG09	2.460	103.640	6.3000	5.3500	5.2800	5.7298
26	TG10	1.460	104.070	9.7500	8.5700	8.7700	9.0368
27	TG19	1.470	103.260	5.6700	5.4300	5.1600	5.6467

**Table 21** Geoid heights from EGM96, OSU91A , Gravimetric and GPS/leveling data

ST. NO	NAME	$\phi$	$\lambda$	$H_{GPS}$	$H_{EGM}$	$H_{OSU}$	$H_{GRAV}$
1	DOP2	1.380	103.610	83.800	84.120	84.080	83.644
2	GP15	2.060	102.560	1.960	2.192	2.622	2.003
3	GP16	2.130	102.730	33.470	33.800	34.170	33.591
4	GP43	2.600	103.780	4.990	5.590	5.530	5.236
5	GP44	2.470	103.070	51.870	52.500	52.770	52.123
6	GP47	2.390	102.930	47.860	48.410	48.730	48.080
7	GP48	1.980	102.930	160.990	161.210	161.530	161.076
8	GP49	1.630	103.200	1.950	2.250	2.540	2.124
9	GP50	1.550	103.400	3.810	4.240	4.400	3.924
10	GP51	1.630	103.670	35.520	35.890	35.850	35.451
11	GP52	1.370	104.270	58.720	58.370	58.220	58.005
12	GP53	1.800	103.900	24.000	24.340	24.250	23.869
13	GP54	1.930	104.090	4.660	4.860	4.740	4.415
14	GP55	2.080	103.890	29.330	29.750	29.720	29.308
15	GP56	2.390	103.870	4.680	5.220	5.130	4.839
16	GP58	2.120	103.430	72.100	72.430	72.680	72.071
17	GP59	1.970	103.230	40.150	40.470	40.770	40.223
18	GP61	2.190	103.190	37.320	37.680	37.990	37.366
19	GP84	1.860	102.940	3.150	3.330	3.640	3.272
20	GP85	1.910	102.740	1.880	2.070	2.410	1.992
21	GP90	1.930	104.110	2.470	2.680	2.550	2.240
22	GP91	1.880	103.690	19.550	19.750	19.800	19.320
23	TG05	2.650	102.920	190.800	191.450	191.740	191.090
24	TG07	1.680	104.070	186.200	185.950	185.790	185.491
25	TG09	2.460	103.640	216.700	217.650	217.720	217.270
26	TG10	1.460	104.070	183.500	184.680	184.480	184.213
27	TG19	1.470	103.260	130.400	130.640	130.910	130.423

**Table 22** Orthometric heights from EGM96, OSU91A, Gravimetric and GPS/leveling data.



ST. NO	NAME	$\phi$	$\lambda$	$H_{GPS}-H_{EGM}$	$H_{GPS}-H_{OSU}$	$H_{GPS}-H_{GRAV}$
1	DOP2	1.380	103.610	-0.320	-0.280	0.156
2	GP15	2.060	102.560	-0.232	-0.662	-0.043
3	GP16	2.130	102.730	-0.330	-0.700	-0.121
4	GP43	2.600	103.780	-0.600	-0.540	-0.246
5	GP44	2.470	103.070	-0.630	-0.900	-0.253
6	GP47	2.390	102.930	-0.550	-0.870	-0.220
7	GP48	1.980	102.930	-0.220	-0.540	-0.086
8	GP49	1.630	103.200	-0.300	-0.590	-0.174
9	GP50	1.550	103.400	-0.430	-0.590	-0.114
10	GP51	1.630	103.670	-0.370	-0.330	0.070
11	GP52	1.370	104.270	0.350	0.500	0.715
12	GP53	1.800	103.900	-0.340	-0.250	0.131
13	GP54	1.930	104.090	-0.200	-0.080	0.245
14	GP55	2.080	103.890	-0.420	-0.390	0.022
15	GP56	2.390	103.870	-0.540	-0.450	-0.159
16	GP58	2.120	103.430	-0.330	-0.580	0.029
17	GP59	1.970	103.230	-0.320	-0.620	-0.073
18	GP61	2.190	103.190	-0.360	-0.670	-0.046
19	GP84	1.860	102.940	-0.180	-0.490	-0.122
20	GP85	1.910	102.740	-0.190	-0.530	-0.112
21	GP90	1.930	104.110	-0.210	-0.080	0.230
22	GP91	1.880	103.690	-0.200	-0.250	0.230
23	TG05	2.650	102.920	-0.650	-0.940	-0.290
24	TG07	1.680	104.070	0.250	0.410	0.709
25	TG09	2.460	103.640	-0.950	-1.020	-0.570
26	TG10	1.460	104.070	-1.180	-0.980	-0.713
27	TG19	1.470	103.260	-0.240	-0.510	-0.023

**Table 23** Absolute orthometric height differences of EGM96, OSU91A and the gravimetric model relative to the local vertical datum.

ID	STN	$h$	$\Delta h$	$H$	$\Delta H$	$N$	$\Delta N$
1	GP58	77.620	<b>fixed</b>	72.100	<b>fixed</b>	5.520	<b>fixed</b>
2	GP15	4.002	73.618	1.960	70.140	2.042	3.478
3	GP16	36.230	41.390	33.470	38.630	2.760	2.760
4	GP43	11.180	66.440	4.990	67.110	6.190	-0.670
5	GP44	55.650	21.970	51.870	20.230	3.780	1.740
6	GP47	51.180	26.440	47.860	24.240	3.320	2.200
7	GP48	164.750	-87.130	160.990	-88.890	3.760	1.760
8	GP49	7.360	70.260	1.950	70.150	5.410	0.110
9	GP50	10.120	67.500	3.810	68.290	6.310	-0.790
10	GP51	42.710	34.910	35.520	36.580	7.190	-1.670
11	GP52	67.810	9.810	58.720	13.380	9.090	-3.570
12	GP53	31.820	45.800	24.000	48.100	7.820	-2.300
13	GP54	12.880	64.740	4.660	67.440	8.220	-2.700
14	GP55	36.740	40.880	29.330	42.770	7.410	-1.890
15	GP56	11.570	66.050	4.680	67.420	6.890	-1.370
16	DOP2	90.910	-13.290	83.800	-11.700	7.110	-1.590
17	GP59	45.180	32.440	40.150	31.950	5.030	0.490
18	GP61	41.830	35.790	37.320	34.780	4.510	1.010
19	GP84	7.130	70.490	3.150	68.950	3.980	1.540
20	GP85	4.930	72.690	1.880	70.220	3.050	2.470
21	GP90	10.780	66.840	2.470	69.630	8.310	-2.790
22	GP91	26.310	51.310	19.550	52.550	6.760	-1.240
23	TG05	193.730	-116.110	190.800	-118.700	2.930	2.590
24	TG07	194.270	-116.650	186.200	-114.100	8.070	-2.550
25	TG09	223.000	-145.380	216.700	-144.600	6.300	-0.780
26	TG10	193.250	-115.630	183.500	-111.400	9.750	-4.230
27	TG19	136.070	-58.450	130.400	-58.300	5.670	-0.150

**Table 24** Relative differences derived from GPS/ leveling data  
(Station GP58 held fixed)

ID	STN	$h$	$\Delta h$	$H$	$\Delta H$	$N$	$\Delta N$
1	GP58	77.620	<b>fixed</b>	72.430	<b>fixed</b>	5.190	<b>fixed</b>
2	GP15	4.002	73.618	2.192	70.238	1.810	3.380
3	GP16	36.230	41.390	33.800	38.630	2.430	2.760
4	GP43	11.180	66.440	5.590	66.840	5.590	-0.400
5	GP44	55.650	21.970	52.500	19.930	3.150	2.040
6	GP47	51.180	26.440	48.410	24.020	2.770	2.420
7	GP48	164.750	-87.130	161.210	-88.780	3.540	1.650
8	GP49	7.360	70.260	2.250	70.180	5.110	0.080
9	GP50	10.120	67.500	4.240	68.190	5.880	-0.690
10	GP51	42.710	34.910	35.890	36.540	6.820	-1.630
11	GP52	67.810	9.810	58.370	14.060	9.440	-4.250
12	GP53	31.820	45.800	24.340	48.090	7.480	-2.290
13	GP54	12.880	64.740	4.860	67.570	8.020	-2.830
14	GP55	36.740	40.880	29.750	42.680	6.990	-1.800
15	GP56	11.570	66.050	5.220	67.210	6.790	-1.600
16	DOP2	90.910	-13.290	84.120	0.000	6.350	-1.160
17	GP59	45.180	32.440	40.470	31.960	4.710	0.480
18	GP61	41.830	35.790	37.680	34.750	4.150	1.040
19	GP84	7.130	70.490	3.330	69.100	3.800	1.390
20	GP85	4.930	72.690	2.070	70.360	2.860	2.330
21	GP90	10.780	66.840	2.680	69.750	8.100	-2.910
22	GP91	26.310	51.310	19.750	52.680	6.560	-1.370
23	TG05	193.730	-116.110	191.450	-119.020	2.280	2.910
24	TG07	194.270	-116.650	185.950	-113.520	8.320	-3.130
25	TG09	223.000	-145.380	217.650	-145.220	5.350	-0.160
26	TG10	193.250	-115.630	184.680	-112.250	8.570	-3.380
27	TG19	136.070	-58.450	130.640	-58.210	5.430	-0.240

**Table 25** Relative differences derived from EGM96 model and GPS/leveling data  
(Station GP58 held fixed)

ID	STN	$h$	$\Delta h$	$H$	$\Delta H$	$N$	$\Delta N$
1	GP58	77.620	<b>fixed</b>	72.680	<b>fixed</b>	4.94	<b>fixed</b>
2	GP15	4.002	73.618	2.622	70.058	1.38	3.56
3	GP16	36.230	41.390	34.170	38.510	2.06	2.88
4	GP43	11.180	66.440	5.530	67.150	5.65	-0.71
5	GP44	55.650	21.970	52.770	19.910	2.88	2.06
6	GP47	51.180	26.440	48.730	23.950	2.45	2.49
7	GP48	164.750	-87.130	161.530	-88.850	3.22	1.72
8	GP49	7.360	70.260	2.540	70.140	4.82	0.12
9	GP50	10.120	67.500	4.400	68.280	5.72	-0.78
10	GP51	42.710	34.910	35.850	36.830	6.86	-1.92
11	GP52	67.810	9.810	58.220	14.460	9.59	-4.65
12	GP53	31.820	45.800	24.250	48.430	7.57	-2.63
13	GP54	12.880	64.740	4.740	67.940	8.14	-3.2
14	GP55	36.740	40.880	29.720	42.960	7.02	-2.08
15	GP56	11.570	66.050	5.130	67.550	6.44	-1.5
16	DOP2	90.910	-13.290	84.080	-11.400	6.83	-1.89
17	GP59	45.180	32.440	40.770	31.910	4.41	0.53
18	GP61	41.830	35.790	37.990	34.690	3.84	1.1
19	GP84	7.130	70.490	3.640	69.040	3.49	1.45
20	GP85	4.930	72.690	2.410	70.270	2.52	2.42
21	GP90	10.780	66.840	2.550	70.130	8.23	-3.29
22	GP91	26.310	51.310	19.800	52.880	6.51	-1.57
23	TG05	193.730	-116.110	191.740	-119.060	1.99	2.95
24	TG07	194.270	-116.650	185.790	-113.110	8.48	-3.54
25	TG09	223.000	-145.380	217.720	-145.040	5.28	-0.34
26	TG10	193.250	-115.630	184.480	-111.800	8.77	-3.83
27	TG19	136.070	-58.450	130.910	-58.230	5.16	-0.22

**Table 26** Relative differences derived from OSU91A model and GPS/leveling data (Station GP58 held fixed)

ID	STN	$h$	$\Delta h$	$H$	$\Delta H$	$N$	$\Delta N$
1	GP58	77.620	<b>fixed</b>	72.071	<b>fixed</b>	5.550	<b>fixed</b>
2	GP15	4.002	73.618	2.003	70.068	1.999	3.551
3	GP16	36.230	41.390	33.591	38.480	2.639	2.911
4	GP43	11.180	66.440	5.236	66.835	5.944	-0.394
5	GP44	55.650	21.970	52.123	19.948	3.527	2.023
6	GP47	51.180	26.440	48.080	23.991	3.100	2.450
7	GP48	164.750	-87.130	161.076	-89.005	3.674	1.876
8	GP49	7.360	70.260	2.124	69.947	5.237	0.313
9	GP50	10.120	67.500	3.924	68.147	6.196	-0.646
10	GP51	42.710	34.910	35.451	36.620	7.260	-1.710
11	GP52	67.810	9.810	58.005	14.066	9.805	-4.255
12	GP53	31.820	45.800	23.869	48.202	7.951	-2.401
13	GP54	12.880	64.740	4.415	67.656	8.465	-2.915
14	GP55	36.740	40.880	29.308	42.763	7.433	-1.883
15	GP56	11.570	66.050	4.839	67.232	6.731	-1.181
16	DOP2	90.910	-13.290	83.644	-11.573	7.266	-1.716
17	GP59	45.180	32.440	40.223	31.848	4.957	0.593
18	GP61	41.830	35.790	37.366	34.705	4.464	1.086
19	GP84	7.130	70.490	3.272	68.799	3.858	1.692
20	GP85	4.930	72.690	1.992	70.079	2.938	2.612
21	GP90	10.780	66.840	2.240	69.831	8.540	-2.990
22	GP91	26.310	51.310	19.320	52.751	6.990	-1.440
23	TG05	193.730	-116.110	191.090	-119.019	2.640	2.910
24	TG07	194.270	-116.650	185.491	-113.420	8.780	-3.230
25	TG09	223.000	-145.380	217.270	-145.199	5.730	-0.180
26	TG10	193.250	-115.630	184.213	-112.142	9.037	-3.487
27	TG19	136.070	-58.450	130.423	-58.352	5.647	-0.097

**Table 27** Relative differences derived from gravimetric model and GPS/leveling data (Station GP58 held fixed)

ID	STN	$\delta N_{EGM}$	$\delta N_{EGM}$	$\delta N_{EGM}$
1	DOP2	-0.152	-0.214	0.159
2	GP15	-0.356	-0.835	-0.216
3	GP16	-0.388	-0.799	-0.205
4	GP43	-0.595	-0.589	-0.149
5	GP44	-0.522	-0.798	-0.239
6	GP47	-0.488	-0.820	-0.242
7	GP48	-0.341	-0.673	-0.128
8	GP49	-0.226	-0.451	0.017
9	GP50	-0.205	-0.350	0.075
10	GP51	-0.243	-0.282	0.100
11	GP52	-0.173	0.027	0.279
12	GP53	-0.313	-0.260	0.094
13	GP54	-0.366	-0.238	0.092
14	GP55	-0.413	-0.364	0.015
15	GP56	-0.523	-0.482	-0.074
16	GP58	-0.410	-0.543	-0.078
17	GP59	-0.349	-0.562	-0.072
18	GP61	-0.426	-0.655	-0.140
19	GP84	-0.299	-0.626	-0.093
20	GP85	-0.309	-0.716	-0.143
21	GP90	-0.367	-0.231	0.096
22	GP91	-0.334	-0.364	0.035
23	TG05	-0.581	-0.916	-0.316
24	TG07	-0.276	-0.156	0.158
25	TG09	-0.539	-0.589	-0.135
26	TG10	-0.197	-0.077	0.219
27	TG19	-0.171	-0.372	0.072

**Table 28** Datum biases of EGM96, OSU91A and gravimetric datums relative to the local vertical datum

ID	STN	$N_{GPS}$	$N_{EGM}^*$	$N_{OSU}^*$	$N_{GRAV}^*$
1	DOP2	7.110	6.942	7.044	7.107
2	GP15	2.042	2.166	2.215	2.215
3	GP16	2.760	2.818	2.859	2.844
4	GP43	6.190	6.185	6.239	6.093
5	GP44	3.780	3.672	3.678	3.766
6	GP47	3.320	3.258	3.27	3.342
7	GP48	3.760	3.881	3.893	3.802
8	GP49	5.410	5.336	5.271	5.220
9	GP50	6.310	6.085	6.07	6.121
10	GP51	7.190	7.063	7.142	7.160
11	GP52	9.090	9.613	9.563	9.526
12	GP53	7.820	7.793	7.83	7.857
13	GP54	8.220	8.386	8.378	8.373
14	GP55	7.410	7.403	7.384	7.418
15	GP56	6.890	6.873	6.922	6.805
16	GP58	5.520	5.6	5.483	5.628
17	GP59	5.030	5.059	4.972	5.029
18	GP61	4.510	4.576	4.495	4.604
19	GP84	3.980	4.099	4.116	3.951
20	GP85	3.050	3.169	3.236	3.081
21	GP90	8.310	8.467	8.461	8.444
22	GP91	6.760	6.894	6.874	6.955
23	TG05	2.930	2.861	2.906	2.956
24	TG07	8.070	8.596	8.636	8.622
25	TG09	6.300	5.889	5.869	5.865
26	TG10	9.750	8.767	8.847	8.818
27	TG19	5.670	5.601	5.532	5.575

**Table 29** Bias corrected geoid heights from EGM96, OSU91A and gravimetric model with GPS/leveling geoid.

ID	STN	$H_{GPS}$	$H_{EGM}^*$	$H_{OSU}^*$	$H_{GRAV}^*$
1	DOP2	83.800	83.968	83.866	83.803
2	GP15	1.960	1.836	1.787	1.787
3	GP16	33.470	33.412	33.371	33.386
4	GP43	4.990	4.995	4.941	5.087
5	GP44	51.870	51.978	51.972	51.884
6	GP47	47.860	47.922	47.910	47.838
7	GP48	160.990	160.869	160.857	160.948
8	GP49	1.950	2.024	2.089	2.141
9	GP50	3.810	4.035	4.050	3.999
10	GP51	35.520	35.647	35.568	35.551
11	GP52	58.720	58.197	58.247	58.284
12	GP53	24.000	24.027	23.990	23.963
13	GP54	4.660	4.494	4.502	4.507
14	GP55	29.330	29.337	29.356	29.323
15	GP56	4.680	4.697	4.648	4.765
16	GP58	72.100	72.020	72.137	71.993
17	GP59	40.150	40.121	40.208	40.151
18	GP61	37.320	37.254	37.335	37.226
19	GP84	3.150	3.031	3.014	3.179
20	GP85	1.880	1.761	1.694	1.849
21	GP90	2.470	2.313	2.319	2.336
22	GP91	19.550	19.416	19.436	19.355
23	TG05	190.800	190.869	190.824	190.774
24	TG07	186.200	185.674	185.634	185.649
25	TG09	216.700	217.111	217.131	217.135
26	TG10	183.500	184.483	184.403	184.432
27	TG19	130.400	130.469	130.538	130.495

**Table 30** Bias corrected orthometric heights from EGM96, OSU91A and gravimetric model with true orthometric height from leveling.



ID	STN	$H_{GPS} - H_{EGM}^*$	$H_{GPS} - H_{OSU}^*$	$H_{GPS} - H_{GRAV}^*$
1	DOP2	-0.168	-0.066	-0.003
2	GP15	0.124	0.173	0.173
3	GP16	0.058	0.099	0.084
4	GP43	-0.005	0.049	-0.097
5	GP44	-0.108	-0.102	-0.014
6	GP47	-0.062	-0.050	0.022
7	GP48	0.121	0.133	0.042
8	GP49	-0.074	-0.139	-0.191
9	GP50	-0.225	-0.240	-0.189
10	GP51	-0.127	-0.048	-0.030
11	GP52	0.523	0.473	0.436
12	GP53	-0.027	0.010	0.037
13	GP54	0.166	0.158	0.153
14	GP55	-0.007	-0.026	0.007
15	GP56	-0.017	0.032	-0.085
16	GP58	0.080	-0.037	0.107
17	GP59	0.029	-0.058	-0.001
18	GP61	0.066	-0.015	0.094
19	GP84	0.119	0.136	-0.029
20	GP85	0.119	0.186	0.031
21	GP90	0.157	0.151	0.134
22	GP91	0.134	0.114	0.195
23	TG05	-0.069	-0.024	0.026
24	TG07	0.526	0.566	0.551
25	TG09	-0.411	-0.431	-0.435
26	TG10	-0.983	-0.903	-0.932
27	TG19	-0.069	-0.138	-0.095

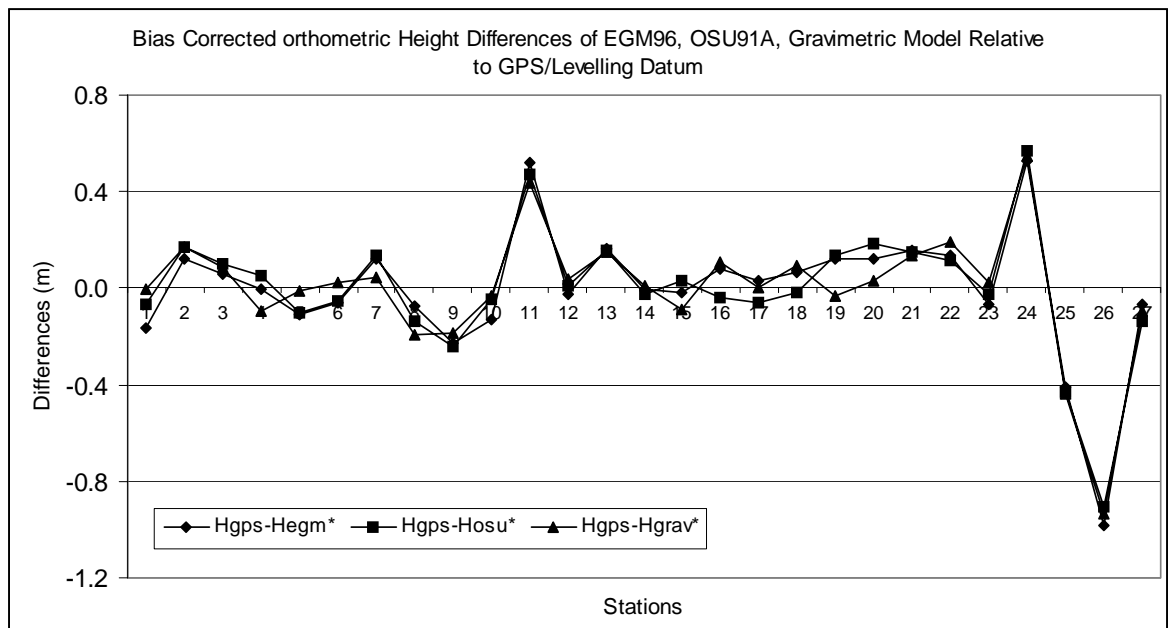
**Table 31** Bias corrected orthometric height differences of EGM96, OSU91A , and gravimetric models relative to the local vertical datum.

ID	STN	$h$	$\Delta h$	$H^*$	$\Delta H^*$	$N^*$	$\Delta N^*$
1	GP58	77.620	<b>fixed</b>	72.020	<b>fixed</b>	5.600	<b>fixed</b>
2	GP15	4.002	73.618	1.836	70.184	2.166	3.434
3	GP16	36.230	41.390	33.412	38.608	2.818	2.782
4	GP43	11.180	66.440	4.995	67.025	6.185	-0.585
5	GP44	55.650	21.970	51.978	20.042	3.672	1.928
6	GP47	51.180	26.440	47.922	24.098	3.258	2.342
7	GP48	164.750	-87.130	160.869	-88.849	3.881	1.719
8	GP49	7.360	70.260	2.024	69.996	5.336	0.264
9	GP50	10.120	67.500	4.035	67.985	6.085	-0.485
10	GP51	42.710	34.910	35.647	36.373	7.063	-1.463
11	GP52	67.810	9.810	58.197	13.823	9.613	-4.013
12	GP53	31.820	45.800	24.027	47.993	7.793	-2.193
13	GP54	12.880	64.740	4.494	67.526	8.386	-2.786
14	GP55	36.740	40.880	29.337	42.683	7.403	-1.803
15	GP56	11.570	66.050	4.697	67.323	6.873	-1.273
16	DOP2	90.910	-13.290	83.968	-11.948	6.942	-1.342
17	GP59	45.180	32.440	40.121	31.899	5.059	0.541
18	GP61	41.830	35.790	37.254	34.766	4.576	1.024
19	GP84	7.130	70.490	3.031	68.989	4.099	1.501
20	GP85	4.930	72.690	1.761	70.259	3.169	2.431
21	GP90	10.780	66.840	2.313	69.707	8.467	-2.867
22	GP91	26.310	51.310	19.416	52.604	6.894	-1.294
23	TG05	193.730	-116.110	190.869	-118.849	2.861	2.739
24	TG07	194.270	-116.650	185.674	-113.654	8.596	-2.996
25	TG09	223.000	-145.380	217.111	-145.091	5.889	-0.289
26	TG10	193.250	-115.630	184.483	-112.463	8.767	-3.167
27	TG19	136.070	-58.450	130.469	-58.449	5.601	-0.001

**Table 32** Bias corrected Relative differences derived from EGM96 model and GPS/leveling data (Station GP58 held fixed)

ID	STN	$h$	$\Delta h$	$H^*$	$\Delta H^*$	$N^*$	$\Delta N^*$
1	GP58	77.620	<b>fixed</b>	72.137	<b>fixed</b>	5.483	<b>fixed</b>
2	GP15	4.002	73.618	1.787	70.350	2.215	3.268
3	GP16	36.230	41.390	33.371	38.766	2.859	2.624
4	GP43	11.180	66.440	4.941	67.196	6.239	-0.756
5	GP44	55.650	21.970	51.972	20.165	3.678	1.805
6	GP47	51.180	26.440	47.910	24.227	3.270	2.213
7	GP48	164.750	-87.130	160.857	-88.720	3.893	1.590
8	GP49	7.360	70.260	2.089	70.048	5.271	0.212
9	GP50	10.120	67.500	4.050	68.087	6.070	-0.587
10	GP51	42.710	34.910	35.568	36.569	7.142	-1.659
11	GP52	67.810	9.810	58.247	13.890	9.563	-4.080
12	GP53	31.820	45.800	23.990	48.147	7.830	-2.347
13	GP54	12.880	64.740	4.502	67.635	8.378	-2.895
14	GP55	36.740	40.880	29.356	42.781	7.384	-1.901
15	GP56	11.570	66.050	4.648	67.489	6.922	-1.439
16	DOP2	90.910	-13.290	83.866	-11.729	7.044	-1.561
17	GP59	45.180	32.440	40.208	31.929	4.972	0.511
18	GP61	41.830	35.790	37.335	34.802	4.495	0.988
19	GP84	7.130	70.490	3.014	69.123	4.116	1.367
20	GP85	4.930	72.690	1.694	70.443	3.236	2.247
21	GP90	10.780	66.840	2.319	69.818	8.461	-2.978
22	GP91	26.310	51.310	19.436	52.701	6.874	-1.391
23	TG05	193.730	-116.110	190.824	-118.687	2.906	2.577
24	TG07	194.270	-116.650	185.634	-113.497	8.636	-3.153
25	TG09	223.000	-145.380	217.131	-144.994	5.869	-0.386
26	TG10	193.250	-115.630	184.403	-112.266	8.847	-3.364
27	TG19	136.070	-58.450	130.538	-58.401	5.532	-0.049

**Table 33** Bias corrected Relative differences derived from OSU91A model and GPS/leveling data (Station GP58 held fixed)



**Fig. 36.** Absolute bias corrected orthometric height differences of EGM96, OSU91A and Gravimetric geoid model relative to local vertical datum